ACCELERATING EXPANSION OF THE UNIVERSE

THESIS SUBMITTED FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
(SCIENCE)
OF
BENGAL ENGINEERING AND SCIENCE UNIVERSITY

BY

WRITAMBHARA CHAKRABORTY

DEPARTMENT OF MATHEMATICS
BENGAL ENGINEERING AND SCIENCE UNIVERSITY
SHIBPUR, HOWRAH - 711 103
INDIA

APRIL, 2009

DECLARATION

This is to certify that the thesis entitled "*ACCELERATING EXPANSION OF THE UNIVERSE*" submitted by Writambhara Chakraborty who got her name registered on 04.11.2006 for the award of Ph.D. (Science) degree of Bengal Engineering and Science University, is absolutely based upon her own work under the supervision of Dr.Ujjal Debnath, Department of Mathematics, Bengal Engineering and Science University, Howrah 711 103, and that neither this thesis nor any part of its has been submitted for any degree / diploma or any other academic award anywhere before.

Ujjal Debnath

(Supervisor)

Lecturer,

Department of Mathematics

Bengal Engineering and Science University

Shibpur

Howrah - 711 103

India

Acknowledgements

To start of I express my sincere gratitude to my supervisor Dr. Ujjal Debnath for his help and support throughout my research work. He has been a true friend, mentor and guide, teaching me all the lessons I needed to learn during my entire research period, sometimes scolding me and sometimes encouraging me to analyse and solve the problems without being impatient. I thank him for all the long but fruitful discussions we used to have regarding all my seven papers with him. I am also grateful to Prof. Subenoy Chakraborty for his valuable suggestions and advice during my research period. He has also coauthored one of my papers. I would like to thank the teachers and staff of the Department of Mathematics, Bengal Engineering and Science University, for their valuable resources and help. I would also like to thank the members of Cosmology and Relativity Centre and the Department of Mathematics, Jadavpur University for their cooperation. Thanks to IUCAA, Pune, India, for their warm hospitality and facilities during my visits there, as a lot of my work were carried out there. I am grateful to Dr. Edmund J. Copeland of University of Nottingham, UK, for his valuable suggestions and immense help during my short period of visit there in 2008. I would like to thank all the teachers and students of the Particle Theory Group and the Astrophysics Group of University of Nottingham for their cooperation during my stay over there.

I would like to thank my colleagues of New Alipore College, Kolkata where I am presently employed as a lecturer. They have been wonderful. Specially I would like to thank my Head of the Department Prof. Dibes Bera and Principal Dr. Sujit Kumar Das, who have helped me a lot by granting me leave whenever I needed. I must mention Prof. B. B. Paira, Director of Heritage Institute of Technology and Dr. Rasajit Kumar

Bera, Head of the Department of Mathematics, Heritage Institute of Technology, where I started of my career as a lecturer and a research student. I am indebted to them for encouraging me and helping me know the right path. I should not forget my colleagues and friends of Heritage Institute of Technology as well, who made that short phase of my life over there a really cherishing one.

I would like to thank Ma and Babai for everything. It is they who taught me all the lessons of life. It is they and their blessing for which I am what I am right now. Thanks to my sister, my parents-in-law and all the other members of my family for their help and support.

Thanks Anirban for being by my side through all the good and bad times in life and constantly encouraging me not to lose hope. You truly are the best friend and partner that I could have ever had. And last but not the least, thanks to my little Rian, whose arrival has truly been the happiest phase transition of my life.

Writambhara Chakraborty
Research Scholar
Department of Mathematics
Bengal Engineering and Science University
Shibpur
Howrah - 711 103
India

Contents

Chapter 1

General Introduction

1.1 Standard Cosmology and FRW Model

The standard Cosmology assumes that at the beginning (approximately 13 billion years ago) there must have been an initial singularity from which the space time suddenly started evolving. Since then the Universe has more or less gone through a process of expansion and cooling from an extremely hot and dense state to the present day cool state. In the first few seconds or so there was a very fast expansion, known as Inflation [http://cosmology.berkeley.edu], which is responsible for the present homogeneous and isotropic Universe. Following this inflationary phase, further expansion cooled down the Universe and matter was produced in the process called baryogenesis. Various light elements like deuterium, helium, lithium-7 were created in a process called Big Bang Nucleosynthesis. The universe was still very hot for the nuclei to bind electrons and form atoms, therefore causing the Universe to be opaque to photons and other electromagnetic radiation. Eventually the temperature drops enough for free nuclei and electrons to combine into atoms in a process called recombination. After the formation of atoms photons could travel freely without being scattered. This caused the emission of Cosmic Microwave Background Radiation (CMBR) which gives us the information about the Universe at that time. Galaxies and stars began to form after a few hundred million of years as a result of gravitational collapse.

Modern Cosmology with the help of observational evidences has within its reach

billions of galaxies and all the heavenly bodies spread all across the vast distances. Advanced observational techniques have strengthened the particular branch of science, sometimes supporting the conventional theories and sometimes producing reverse results. As a consequence of these observational advances cosmology has become more or less data driven, so that all the theories need to be fitted with array of observations, although there are still doubts and debates about the reliability and interpretation of such data [Frieman, 1994].

The standard cosmological model which is very successful in describing the evolution of the Universe, is based on homogeneous and isotropic Friedmann-Robertson-Walker (FRW) spacetime. Homogeneity and isotropy that we assume for this model is supported by strong observational data [Smoot etal, 1992; Maddox etal, 1990; Collins etal, 1992], CMBR measurements and galaxy redshift surveys [Fisher etal, 1993; Geller etal, 1989]. This global isotropy and homogeneity which is the foundation of the standard cosmology is called Cosmological Principle. Cosmological Principle leads to Hubble's Law, which says that the recession velocity of galaxy is proportional to the distance from us, i.e., $V = HD$. The proportionality constant $H = \frac{\dot{a}}{a}$ is known as Hubble constant and $a(t)$ is the scale factor.

We now look at Einstein's model of the Universe. In 1932, Einstein and de Sitter presented the Standard Cosmological Model of the Universe, which has been the most favourite among the cosmologists till 1980.

Initially Einstein assumed homogeneity and isotropy in his cosmological problem. He chose a time coordinate t such that the line element of static space-time could be described by [Narlikar, *An Introduction to Cosmology*],

$$ds^2 = c^2dt^2 - g_{\mu\nu}dx^\mu dx^\nu \tag{1.1}$$

where $g_{\mu\nu}$ are functions of space coordinates $x^\mu(\mu, \nu = 1, 2, 3)$ only.

We can now construct the homogeneous and isotropic closed space of three dimensions that Einstein wanted for his model of the Universe. The equation of such a 3-surface of a four dimensional hypersphere of radius a is given in Cartesian coordinates x_1, x_2, x_3, x_4 by

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = a^2 \tag{1.2}$$

Therefore the spatial line element on the surface is given by

$$d\sigma^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 + (dx_4)^2 = a^2[d\chi^2 + sin^2\chi(d\theta^2 + sin^2\theta d\phi^2)] \tag{1.3}$$

where $x_1 = a\ sin\chi\ cos\theta,\ x_2 = a\ sin\chi\ sin\theta\ cos\phi,\ x_3 = a\ sin\chi\ sin\theta\ sin\phi,\ x_4 = a\ cos\chi$ and the ranges of $\theta,\ \phi$ and χ are given by $0 \leq \chi \leq \pi,\ 0 \leq \theta \leq \pi,\ 0 \leq \phi \leq 2\pi$

Another way to express $d\sigma^2$ through coordinates $r,\ \theta,\ \phi$ with $r = sin\chi$, ($0\leq r \leq 1$) is,

$$d\sigma^2 = a^2\left[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + sin^2\theta d\phi^2))\right] \tag{1.4}$$

The line element for the Einstein Universe is therefore given by

$$ds^2 = c^2dt^2 - d\sigma^2 = c^2dt^2 - a^2\left[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + sin^2\theta d\phi^2))\right] \tag{1.5}$$

This line element is for + ve curvature only.

In general we have

$$ds^2 = c^2dt^2 - a^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{1.6}$$

where, $k = 0, +1, -1$ for zero, +ve, -ve curvatures respectively and are also known as flat, closed, open models and $a(t)$ is known as the scale factor or expansion factor.

Thus for $c = 1$, FRW line element reduces to,

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right) \tag{1.7}$$

Now the energy-momentum tensor describing the material contents of the Universe is given by

$$T_{\mu\nu} = (\rho c^2 + p)u_\mu u_\nu - p g_{\mu\nu} \tag{1.8}$$

where, $\rho = T_{00}$ is mean energy density of matter, $p = T_{ii}$ is the pressure, and $u_\mu = (c, 0, 0, 0)$ is fluid four velocity. Usually $\Omega_i = \frac{\rho_i}{\rho_c}$, where $\rho_c = 3H^2/8\pi G$, is called the critical energy density.

Also the equation of motion describing the Universe, known as Einstein field equations in general relativity are

$$G_{ik} = R_{ik} - \frac{1}{2}g_{ik}R = \frac{8\pi G}{c^4}T_{ik} \tag{1.9}$$

where, G_{ik} = Einstein Tensor, R_{ik} = Ricci Tensor, R = Ricci Scalar.

Thus for a static ($\dot{a} = 0$), dust filled ($p = 0$) and closed ($k = +1$) model of the universe, the field equations yield, (choosing $8\pi G = c = 1$)

$$\frac{3}{a^2} = \rho,\ \frac{1}{a^2} = 0 \tag{1.10}$$

Clearly no feasible solution is possible from these equations, thus suggesting that no static homogeneous isotropic and dense model of the Universe is possible under the

regime of Einstein equations stated above.

For this reason Einstein later modified his field equations as

$$G_{ik} = \frac{8\pi G}{c^4} T_{ik} + \Lambda g_{ik} \tag{1.11}$$

Thus introducing the famous Λ-term, known as Cosmological Constant. With this, the picture changes to, $\frac{3}{a^2} = \rho + \Lambda,\ \frac{1}{a^2} = \Lambda$, so that $a = \frac{1}{\sqrt{\Lambda}} = \sqrt{\frac{2}{\rho}}$.

This Λ is universal constant like $G,\ c$, etc. To estimate the value of Λ the value of ρ was used in 1917, which are given as follows,

$a \sim 10^{29}\ cm,\ \ \rho \sim 10^{-31}\ gm\ cm^{-3},\ \ \Lambda \sim 10^{-58}\ cm^{-2}$.

The Λ - term introduces a force of repulsion between two bodies that increases in proportion to the distances between them.

Einstein first proposed the cosmological constant as a mathematical fix to the theory

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{1}{3}\rho \tag{1.12}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) \tag{1.13}$$

in general relativity. In its simplest form, general relativity predicted that the universe must either expand or contract. Einstein thought the universe was static, so he added this new term to stop the expansion. Friedmann, a Russian mathematician, realized that this was an unstable fix and proposed an expanding universe model called **Friedmann model** of the Universe.

For expanding Universe, $\dot{a} > 0$. But since for normal matter $\rho > 0,\ p \geq 0$, hence the second equation gives $\ddot{a} < 0$. So that, $\dot{a}$ is decreasing, i.e., expansion of Universe is decelerated. This model is known as **Standard Cosmological Model** (SCM).

Teams of prominent American and European Scientists using both Hubble Space Telescope (HST) and Earth based Telescopes had announced in 1998 the results of their many years observation and measurement of the expansion of the Universe. Their collective announcement was that the Universe is not just expanding, it is in fact, expanding with ever increasing speed. This combined discovery has been a total surprise for Cosmology! The SCM states that Universe is decelerating but recent high redshift type Ia Supernovae (explosion) observation suggests the Universe is accelerating [Perlmutter et al, 1998, 1999; Riess et al, 1998; Garnavich et al, 1998]. So there must be some matter field, either neglected or unknown, which is responsible for accelerating Universe.

For accelerating Universe, $\ddot{a} > 0$, i.e., $\rho + 3p < 0$, i.e., $p < -\frac{\rho}{3}$. Hence, the matter has the property −ve pressure. This type of matter is called Quintessence matter (Q-matter) and the problem is called Quintessence problem. The missing energy in Quintessence problem can be associated to a dynamical time dependent and spatially homogeneous / inhomogeneous scalar field evolving slowly down its potential $V(\phi)$. These types of cosmological models are known as quintessence models. In this models the scalar field can be seen as a perfect fluid with a negative pressure given by $p = \gamma\rho$, $(-1 \leq \gamma \leq 1)$.

Introducing Λ term,

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{1}{3}\rho + \frac{1}{3}\Lambda = \frac{1}{3}(\rho + \Lambda) \tag{1.14}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) + \frac{1}{3}\Lambda = -\frac{1}{6}[(\rho + \Lambda) + 3(p - \Lambda)] \tag{1.15}$$

If Λ dominates, $\ddot{a} > 0$, i.e, Universe will be accelerating.

For normal fluid, relation between p and ρ is given by, $p = \gamma\rho$, $(0 \leq \gamma \leq 1)$ which is called equation of state.

For dust, $\gamma = 0$; for radiation, $\gamma = \frac{1}{3}$.

Here Λ satisfies an equation of state $p = -\rho$, so pressure is negative. Therefore for accelerating Universe we need such type of fluid which generates negative pressure. The most puzzling questions that remain to be explained in cosmology are the questions about the nature of these types of matter or the mystery of "missing mass", that is, the "dark energy" and "dark matter" problem.

1.2 Dark Energy and Dark Matter

In 1998, two teams studying distant Type Ia supernovae presented independent results of observation that the expansion of the Universe is accelerating [Perlmutter et al, 1998, 1999; Riess et al, 1998; Garnavich et al, 1998]. For the last few decades cosmologists had been trying to measure the deceleration of the Universe caused by the gravitational attraction of matter, characterized by the deceleration parameter $q = -\frac{a\ddot{a}}{\dot{a}^2}$, as suggested by SCM. Therefore, discovery of cosmic acceleration has been proved to be one of the most challenging and important development in Cosmology, which evidently indicates existence of some matter field, either unknown or neglected so far, responsible for this accelerated Universe [Bachall et al, 1999].

Observations suggest that $\Omega_{baryon} \simeq 0.04$ and $\Omega_{total} = 1.02 \pm 0.02$. That is ordinary matter or baryons (atoms) described by the standard model of particle Physics is only approximately 4% of the total energy of the Universe, another 23% is some **dark matter** and 73% is **dark energy**, yet to be discovered. Thus dark matter and dark energy are considered to be the missing pieces in the cosmic jigsaw puzzle [Sahni, 2004]

$\Omega_{total} - \Omega_{baryons} =?$

Although the nature of neither dark matter (DM) nor dark energy (DE) is currently known, it is felt that both DM and DE are non-baryonic in nature. Non-baryonic dark matter does not emit, absorb or scatter light and also it has negligible random motion [Springel et al, Bennett, 2006]. Thereafter it is called cold dark matter (CDM). Although, depending on the mass of the particle it is usually differentiated between hot dark matter and cold dark matter. Non-baryonic Hot Dark Matter (HDM) particles are assumed to be relativistic while decoupling from the masses around and so have a very large velocity dispersion (hence called hot), whereas, CDM particles have a very small velocity dispersion and decouple from the masses around when they are non-relativistic. Non-baryonic dark matters do not seem to interact with light or any known baryonic matter, therefore they are called weakly interacting massive particles (WIMP). The standard cold dark matter (SCDM) paradigm assumes $\Omega_m = 1$, whereas, LCDM, a modification of SCDM, which consists of CDM and a Lambda-term or cosmological constant $\Omega_m = 0.3$ and the hot dark matter scenario is constrained due to structure formation calculations. A variety of dark matter candidates are present in literature, among which a WIMP called nutralino is one of the most favoured one. This is a 100-1000 GeV particle and fermionic partner to the gauge and Higgs bosons [Sahni, 2004]. Another candidate for CDM is an ultra-light pseudo-Goldstone boson called an axion, which arises in the solution of the strong CP problem of particle physics [Masso, 2003]. Other candidates for CDM are string theory motivated modulii fields [Brustein, 1998]; non-thermally produced superheavy particles having mass $\sim 10^{14}$ GeV and dubbed Wimpzillas [Maartens, 2003]; axino (with a mass m $\sim$ 100keV and a reheating temperature of 106K) and gravitino, superpartners of the axion and graviton respectively [Roszkowski, 1999]. Although dark matter is assumed to be comprised of particles which are pressureless cold dark matter with an equation of state $\omega_{CDM} = 0$, nature of dark matter is yet to be discovered.

Existence of dark energy has been driven by the recently discovered accelerated expansion of the Universe. Equation (1.13) shows that acceleration of the Universe is to be

expected when pressure is sufficiently negative. Also expansion of the Universe should decelerate if it is dominated by baryonic matter and CDM. Two independent groups observed SNe Ia to measure the expected deceleration of the Universe in 1998. Instead, both the groups surprisingly discovered that the expansion of the Universe is accelerating. This discovery hinted at a new negative pressure contributing to the mass-energy of the Universe in equation (1.13). Now, equation (1.13) that a universe consisting of only a single component will accelerate if $\omega < -1/3$. Fluids satisfying $\rho + 3p \geq 0$ or $\omega \geq -1/3$ are said to satisfy the **strong energy condition** (SEC). We therefore find that, in order to accelerate, dark energy must violate the SEC. Another condition is the **weak energy condition** (WEC) $\rho + p \geq 0$ or $\omega \geq -1$. If WEC is not satisfied, Universe can expand faster than the exponential rate causing cosmic *Big Rip*.

Now the greatest challenge for cosmologists was to find an explanation for this accelerated expansion of the Universe. One modification was made by Einstein himself in1917 by introducing the cosmological constant term, to act as a gravitational repulsive term, to achieve a static Universe, as seen in the previous section. Later it was dropped after Hubble's discovery of the expansion of the Universe in 1929. In some models of the dark energy, it is identified with this cosmological constant Λ. However, particle physics looks at cosmological constant as an energy density of the vacuum. Moreover, the energy scale of Λ should be much larger than that of the present Hubble constant H_0, if it originates from the vacuum energy density. This is the cosmological constant problem [Weinberg, 1989].

Thus cosmological constant with equation of state $\omega = -1$ may play the role to drive the recent cosmic acceleration. Another possibility is that there exists a universal evolving scalar field with equation of state $\omega < -1/3$, called quintessence field. Also a few more models have been proposed recently in support of dark energy analysis. We discuss a few of these models below.

1.2.1 Cosmological Constant

Cosmological constant is the simplest form of dark energy ($\omega = -1$). Also several cosmological observations suggest that cosmological constant should be considered as the most natural candidate for dark energy [Padmanabhan, 2006]. As discussed above Einstein [1917] introduced cosmological constant Λ in order to make the field equations of GR compatible with Mach's Principle and thus producing a static Einstein Universe. Later in 1922, Friedmann constructed a matter dominated expanding universe without a cosmological constant [Sahni etal, 2000]. Friedmann cosmological model was accepted as a standard cosmological model after Hubble's discovery of expansion of Universe [Weinberg, 1989] and thus Einstein dropped the idea of a static Universe and henceforth positive Λ-term. Later Bondi [1960] and McCrea [1971] realized that if the energy density of the cosmological constant is comparable to the energy density of the present epoch, the cosmological model takes a very reliable form [Padmanabhan, 2003]. But the importance of cosmological constant was first noticed when Zeldovich [1968] showed that zero-point vacuum fluctuations have Lorentz invariant of the form $p_{vac} = -\rho_{vac}$, which is the equation of state for Λ, therefore $T^{vac}_{\mu\nu} = \Lambda g_{\mu\nu}$, which shows that the stress-energy of vacuum is mathematically equivalent to Λ.

In 1970, the discovery of supersymmetry became a turning point involving the cosmological constant problem, as the contributions to vacuum energy from fermions and bosons cancel in a supersymmetric theory. However supersymmetry is expected to be broken at low temperatures at that of the present Universe. Thus cosmological constant is expected to vanish in the early universe and exist later when the temperature is sufficiently low. Which poses a new problem regarding cosmological constant as a large value of Λ at an early time is useful from the viewpoint of inflation, whereas a very small current value of Λ is in agreement with observations [Sahni etal, 2000]. A positive Λ term was still of interest as the inflationary models constructed during 1970 and 1980's

demanded matter, in the form of vacuum polarization or minimally coupled scalar field, behaving as a weakly time-dependent Λ-term. However, recently constructing de-Sitter vacua in string theory or supergravity has been very useful in solving cosmological constant problem [Copeland etal, 2006].

Now introducing Λ in Einstein field equations of GR (1.9), we get,

$$G_{ik} + \Lambda g_{ik} = 8\pi G T_{ik} \tag{1.16}$$

Thus the Friedmann equations become,

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \tag{1.17}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{1.18}$$

and the energy conservation law reads,

$$d(\rho a^3) = -p da^3 \tag{1.19}$$

which on further reduction gives,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \tag{1.20}$$

Now we define [Carroll etal, 1992],

$$\Omega_\rho = \frac{8\pi G}{3{H_0}^2}\rho_0,\ \Omega_\Lambda = \frac{\Lambda}{3{H_0}^2},\ \Omega_k = -\frac{k}{{H_0}^2{a_0}^2} \tag{1.21}$$

Thus equation (1.17) in combination with equation (1.21) gives,

$$\Omega_\rho + \Omega_\Lambda + \Omega_k = 1 \tag{1.22}$$

and, we assume,

$$\Omega_\rho + \Omega_\Lambda = \Omega_{tot} \tag{1.23}$$

so that, $\Omega_{tot} < 1$ (> 1) implies a spatially open (closed) Universe.

Now as we have seen before,

$$p_{vac} = -\rho_{vac} \tag{1.24}$$

and thus the relation between Λ and vacuum energy density is,

$$\Lambda = 8\pi G \rho_{vac} \tag{1.25}$$

Thus the deceleration parameter ($q = -\frac{a\ddot{a}}{\dot{a}^2}$) becomes,

$$q = \frac{(1+3\omega)}{2}\Omega_\rho - \Omega_\Lambda \tag{1.26}$$

so that, for the present universe consisting of pressureless dust (dark matter) and Λ, the deceleration parameter takes the form,

$$q_0 = \frac{1}{2}\Omega_\rho - \Omega_\Lambda \tag{1.27}$$

Recent cosmological observations also suggest the existence of a non-zero cosmological constant. One of the most reliable observation is that of high redshift Type Ia supernovae [Perlmutter et al 1998, Riess et al 1998], which hints at a recently accelerating Universe with a large fraction of the cosmological density in the form of a cosmological constant. Also models based on the cold dark matter model (CDM) with $\Omega_m = 1$ does not meet up the values obtained from cosmological and CMBR observations, whereas, a flat low density CDM+Λ universe (ΛCDM) with $\Omega_m \sim 0.3$ and $\Omega_\Lambda \sim 0.7$, such that $\Omega_{tot} \approx 1$ agrees remarkably well with a wide range of observational data ranging from

large and intermediate angle CMB anisotropies to observations of galaxy clustering on large scales [Sahni etal, 2000]. This is evidence enough for a non-zero Λ-term.

Now, the vacuum energy density ρ_{vac}, proportional to Λ, obeys [Cohn, 1998], $\frac{\rho_{vac}^{observed}}{\rho_{vac}^{observed}} < 10^{-52}$

This disparity between the expected and the observed value of Λ poses the famous cosmological constant problem.

One solution to this problem of very large Λ-term (predicted by field theory) and a small one (suggested by observations) is to make Λ time-dependent. An initial large value of Λ would explain inflation and galaxy formation, while subsequent slow decay of $\Lambda(t)$ would produce a small present value $\Lambda(t_0)$ to be reconciled with observations suggesting $\Omega_\Lambda \sim 0.7$ [Sahni etal, 2000]. For this purpose a few phenomenological models have been introduced, which Sahni has classified into three categories [Sahni, 2000], viz, (1) *Kinematic models* where Λ is a function of cosmic time t or the scale factor $a(t)$ in FRW cosmology; (2) *Hydrodynamic models* where Λ is described by a barotropic fluid with some equation of state and (3) *Field-theoretic models* where Λ is assumed to be a new physical classical field with some phenomenological Lagrangian.

Now keeping Λ to be time-dependent and moving the cosmological term on the right hand side of equation (1.16) [Overduin and Cooperstock, 1998], we have,

$$G_{ik} = 8\pi G\tilde{T}_{ik} \tag{1.28}$$

where,

$$\tilde{T}_{ik} = T_{ik} - \frac{\Lambda}{8\pi G}g_{ik} \tag{1.29}$$

This implies that the effective energy-momentum tensor is described by effective pressure $\tilde{p} = p - \frac{\Lambda}{8\pi G}$ and effective energy-density $\tilde{\rho} = \rho + \frac{\Lambda}{8\pi G}$. Thus the energy conservation law (1.19) becomes,

$$d\left[(\rho + \frac{\Lambda}{8\pi G})a^3\right] = -(p - \frac{\Lambda}{8\pi G})da^3 \tag{1.30}$$

along with equation (1.17) and (1.18).

A few phenomenological models of time-variant Λ-term are given in table I followed by the one presented by Overduin and Cooperstock.

Table I

Λ-decay law	Reference
$\Lambda \propto H^2$	Lima and Carvalho (1994); Wetterich (1995); Arbab (1997)
$\Lambda \propto \frac{\ddot{a}}{a}$	Arbab (2003, 2004); Overduin and Cooperstock (1998)
$\Lambda \propto \rho$	Viswakarma (2000)
$\Lambda \propto t^{-2}$	Endo and Fukui (1977); Canuto, Hsieh and Adams (1977); Bertolami (1986); Berman and Som (1990); Beesham (1994); Lopez and Nanopoulos (1996); Overduin and Cooperstock (1998)
$\Lambda \propto t^{-\alpha}$ (α being a constant)	Kalligas, Wesson and Everitt (1992, 1995); Beesham (1993)
$\Lambda \propto a^{-2}$	Ozer and Taha (1986, 1987); Gott and Rees (1987);Kolb (1989) Chen etal (1990); Abdel-Rahman (1992); Abdussattar etal (1997)
$\Lambda \propto a^{-\alpha}$ α being a constant	Olson and Jordan (1987); Pavon (1991); Maia and Silva (1994) Silveira and Waga (1994, 1997); Torres and Waga (1996)
$\Lambda \propto f(H)$	Lima and Maia (1994); Lima and Trodden (1996)

The Λ-decay laws that have been analyzed in the theory are mostly based on scalar fields or derived from the modified version of the Einstein action principle and they

show the decay of the cosmological term is consistent with the cosmological observations. Many of these works do not exhibit the energy transfer between the decaying Λ-term and other forms of matter [Ratra and Peebles, 1988]. Some models overlook this energy-exchange process assuming that equal amounts of matter and antimatter are being produced (if the decay process does not violate the baryon number)[Freese etal, 1987]. These models are constrained by diffuse gamma-ray background observations [Matyjasek, 1995]. Some models refer to this energy exchange process as production of baryons or radiation. These models are constrained by CMB anisotropies [Silveira and Waga, 1994, 1997] and nucleosynthesis arguments [Freese etal, 1987]. Irrespective of the sources, these models (many of which are independently motivated), in general, solve the cosmological problems in a simpler way and satisfy the cosmological observational constraints [Overduin and Cooperstock, 1998].

Some authors have incorporated a variable gravitational constant also while retaining the usual energy conservation law [Arbab, 1997, 1998, 2002]. A possible time-variable G was suggested by Dirac [1988] on the basis of his large number hypothesis and since then many workers have extended GR with $G = G(t)$ to obtain a satisfactory explanation for the the present day acceleration [Abdel-Rahman, 1990; Abbussattar and Vishwakarma, 1997; Kalligas etal, 1992]. Since G couples geometry to matter, a time-dependent G as well as a time-dependent Λ can explain the evolution of the Universe, as the variation of G cancels the variation of Λ, thus retaining the energy conservation law. There also have been works considering G and Λ to be coupled scalar fields. This approach is similar to that of Brans-Dicke theory [Brans and Dicke, 1961]. This keeps the Einstein field equations formally unchanged [Vishwakarma, 2008].

In FRW model the Einstein field equations with variable G and Λ

$$G_{\mu\nu} + \Lambda(t) g_{\mu\nu} = 8\pi G(t) T_{\mu\nu} \tag{1.31}$$

take the forms [Abdel-Rahman, 1990]:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \tag{1.32}$$

$$2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} = -8\pi G p + \Lambda \tag{1.33}$$

Elimination of $\ddot{a}$ and k gives,

$$\dot{\rho} + 3H(\rho + p) + \frac{\dot{\Lambda}}{8\pi G} + \frac{\dot{G}}{G}\rho = 0 \tag{1.34}$$

Assuming that the usual conservation law holds we can split this equation as,

$$\dot{\Lambda} + 8\pi \dot{G}\rho = 0 \tag{1.35}$$

and equation (1.20).

Equation (1.35) represents a coupling between vacuum and gravity [Arbab, 2001]. This also shows that gravitation interaction increases as Λ decreases and thus causing the accelerated expansion of the Universe, i.e., while Λ decreases with time, the gravitational constant is found to increase with time, causing the Universe to have accelerated expansion in order to overcome the increasing gravity [Arbab, 1999]. Arbab has commented in his paper that this big gravitational force could have been the reason to stop inflation and thereafter creation of matter in the early Universe by forcing the small particles to form big ones. Unlike the Dirac model, this model guarantees the present isotropic Universe as the anisotropy decreases as G increases with time. In a closed Universe with variable Λ and G, Abdel-Rahman has shown that $a \propto t$, $G \propto t^2$ and $\rho \propto t^{-4}$ in the radiation dominated era. Again Kalligas etal have obtained a static solution with variable G and Λ. Berman, Abdel-Rahman and Arbab have independently obtained the solution $a \propto t$ in both matter and radiation dominated era. Berman and Arbab have

independently remarked that $\Lambda \propto t^{-2}$ plays an important role in evolution of the Universe. Varying G theories have also been studied in the context of induced gravity model where G is generated by means of a non-vanishing vacuum expectation value of a scalar field [Zee, 1979; Smolin, 1979; Adler, 1980; Vishwakarma, 2008]. Another theory uses a renormalization group flow near an infrared attractive fixed point [Bonanno and Reuter, 2002; Vishwakarma, 2008]. Mostly the varying G models are consistent with redshift SNe Ia observations. Some of these models solve the horizon and flatness problem also without any unnatural fine tuning of the parameters [Bonanno and Reuter, 2002].

1.2.2 Quintessence Scalar Field

The fine tuning problem facing dark energy models with a constant equation of state can be avoided if the equation of state is assumed to be time dependent. An important class of models having this property is scalar quintessence field proposed by Wetterich [1988] and Ratra and Peebles [1988] which slowly rolls down its potential such that the potential term dominates over the kinetic term and thus generates sufficient negative pressure to drive the acceleration. This Q-field couple minimally to gravity so that the action for this field is given by

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) \tag{1.36}$$

where, $V(\phi)$ is the potential energy and the field ϕ is assumed to be spatially homogeneous. Thus the energy-momentum tensor is given by, [Copeland etal,2006],

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right) \tag{1.37}$$

For a scalar field ϕ, with Lagrangian density $\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$ in the background of flat FRW Universe, we have the pressure and energy density are respectively,

$$p = -T^\mu_\mu = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{1.38}$$

$$\rho = T^0_0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{1.39}$$

Hence the equation of state (EOS) parameter is,

$$\omega = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{1.40}$$

where an overdot means derivative with respect to cosmic time and prime denotes differentiation w.r.t. ϕ. Thus if $\dot{\phi}^2 << V(\phi)$, that is, Q-field varies very slowly with time, it behaves as a cosmological constant, as $\omega \approx -1$, with $\rho_{VAC} \simeq V[\phi(t)]$.

Now the equation of motion for the quintessence field is given by the Klein-Gordon equation,

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0 \tag{1.41}$$

From equation (1.40) we see that ω can take any value between -1 (rolling very slowly) and $+1$ (evolving very rapidly) and varies with time [Frieman etal, 2008]. Also, $\omega < -1/3$ if $\dot{\phi}^2 < V(\phi)$ and $\omega < -1/2$ if $\dot{\phi}^2 < \frac{2}{3}V(\phi)$.

Various quintessence potentials have been introduced in order to explain recent cosmic acceleration. *Inverse Power Law Potential*, given by $V(\phi) = V_0\phi^{-\alpha}$ is one of the simplest of the lot [Ratra etal, 1988] . *Exponential Potential* [Wetterich, 1995] given by, $V(\phi) = V_0\ exp(-\frac{\sqrt{8\pi}\alpha\phi}{M_P})$, where M_P is the reduced Planck mass, is one of the most favoured one among cosmologists. But in this case $\frac{\rho_\phi}{\rho_{total}} < 0.02$, suggesting that exponential potential cannot make the transition from subdominant to dominant energy density component of the universe in late times. Sahni and Wang proposed a model in 2000 as $V(\phi) = V_0(cosh\ \alpha\phi - 1)^p$. This model interpolates from $V \propto\ exp(p\alpha\phi)$ to $V \propto (\alpha\phi)^{2p}$, thereby preserving some of the properties of the simpler exponential potential but allowing a different late time behavior. This potential describes quintessence for $p \leq 1/2$ and pressureless CDM for $p = 1$. Thus the cosine hyperbolic potential is able

to describe both dark matter and dark energy within a tracker framework [Sahni etal, 2000]. Sahni [2004] has presented a few quintessence potentials proposed in literature in a tabular form as below:

Table II

Quintessence Potential	Reference
$V_0 \exp(-\lambda\phi)$	Ratra and Peebles (1988), Wetterich (1988), Ferreira and Joyce (1998)
$m^2\phi^2$, $\lambda\phi^4$	Frieman etal (1995)
V_0/ϕ^α, $\alpha > 0$	Ratra and Peebles (1988)
$V_0 \exp(\lambda\phi^2)$	Brax and Martin (1999, 2000)
$V_0(\cosh \alpha\phi - 1)^p$	Sahni and Wang (2000)
$V_0 \sinh^{-\alpha}(\lambda\phi)$	Sahni and Starobinsky (2000), Ureña-López and Matos (2000)
$V_0(e^{\alpha\kappa\phi} + e^{\beta\kappa\phi})$	Barreiro, Copeland and Nunes (2000)
$V_0(expM_P/\phi - 1)$	Zlatev, Wang and Steinhardt (1999)
$V_0[(\phi - B)^\alpha + A]e^{-\lambda\phi}$	Albrecht and Skordis (2000)

In order to obtain a feasible dark energy model, the energy density of the scalar field should be subdominant during the radiation and matter dominating eras, emerging only at late times to give rise to the current observed acceleration of the universe [Copeland etal, 2006]. Therefore, we introduce a barotropic fluid in the background, with EOS given by, $\omega_m = \frac{p_m}{\rho_m}$, where p_m and ρ_m are the pressure and energy density of the barotropic fluid respectively.

A homogeneous and isotropic universe is characterized by the Friedmann-Robertson-Walker (FRW) line element is given by equation (1.7) with $c = 1$. Thus, the over all stress-energy tensor of the scalar field ϕ in presence of barotropic fluid the has the form,

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \qquad u_\mu u^\mu = -1 \tag{1.42}$$

where $\rho = \rho_m + \rho_\phi$ and $p = p_m + p_\phi$. Here ρ_ϕ and p_ϕ are the energy density and pressure of the Q-field given by equations (1.39) and (1.38) respectively. If ω_m is assumed to be a constant, the fluid energy will be given by $\rho_m = \rho_0 a^{-3(1+\omega_m)}$ and ω_ϕ dynamically changes in general.

The Friedmann equations together with the energy conservation of the normal matter fluid and quintessence (Klein-Gordon equation) are,

$$H^2 + \frac{k}{a^2} = \frac{1}{3}(\rho_m + \rho_\phi) \tag{1.43}$$

$$\dot{\rho}_m + 3H\gamma_m\rho_m = 0 \tag{1.44}$$

together with equation (1.41).

where $H \equiv \frac{\dot{a}}{a}$ denotes the Hubble factor. Introducing $\Omega_m \equiv \frac{\rho_m}{\rho_c}$, $\Omega_\phi \equiv \frac{\rho_\phi}{\rho_c}$, $\Omega_k = -\frac{k}{(aH)^2}$ and $\Omega = \Omega_m + \Omega_\psi$ with $\rho_{_c} \equiv 3H^2$ as the critical density, Ellis et al [1997] showed that the matter violates the strong energy condition $\rho + 3p \leq 0$ and so the deceleration parameter $q = -\frac{a\ddot{a}}{\dot{a}^2} < 0$. Hence as a consequence the universe accelerates its expansion.

In the investigation of cosmological scenarios we are also interested about those solutions in which the energy density of the scalar field mimics the background fluid energy density, called *scaling solutions* [Copeland etal, 1998]. Exponential potentials have been proved to give rise to scaling solutions and so can play an important role in quintessence scenarios, allowing the field energy density to mimic the background being sub-dominant during radiation and matter dominating eras. In this case, as long as the scaling solution is the attractor, then for any generic initial conditions, the field would sooner or later enter the scaling regime, thereby opening up a new line of attack on the fine tuning

problem of dark energy [Copeland etal, 2006].

Going on with the analysis, very interesting question is whether it is possible to construct a successful common scheme for the two cosmological mechanisms involving rolling scalar fields i.e., quintessence and inflation. This perspective has the appealing feature of providing a unified view of the past and recent history of the universe, but can also remove some weak points of the two mechanisms when considered separately. In general, scalar fields tend to be of heavy (high energy). When renormalized, scalar fields tend to become even heavier. This is acceptable for inflation, because Universe was in a very high energy state at that epoch, but it seems highly unnatural for the recent Universe [Bennet, 2006]. Inflation could provide the initial conditions for quintessence without any need to fix them by hand and quintessence could hope to give some more hints in constraining the inflation potential on observational grounds.

From theoretical point of view a lot of works [Caldwell et al, 1998; Ostriker et al, 1995; Peebles, 1984; Wang et al, 2000; Perlmutter et al, 1999; Dodelson et al, 2000; Faraoni, 2000] have been done to solve the quintessence problem. Scalar fields [Peebles et al, 1988, 2002; Ratra et al, 1988; Ott, 2001; Hwang et al, 2001; Ferreira et al, 1998] with a potential giving rise to a negative pressure at the present epoch, a dissipative fluid with an effective negative stress [Cimento et al, 2000] and more exotic matter like a frustrated network of non-abelian cosmic strings or domain wall [Bucher et al, 1999; Battye et al, 1999], scalar fields with non-linear kinetic term, dubbed K-essence model [Armendariz-Picon etal, 2001], are plausible candidates of Q-matter. Also, there exist models of quintessence in which the quintessence field is coupled to dark matter and/or baryons [Amendola, 2000].

Scalar fields, although, being very popular in theory, they have several shortcomings, as they need some suitable potential to explain the accelerated expansion, they need

to assume cosmological constant to be zero [Padmanabhan, 2006]. Also, most of the fields (Q-matter field, tracker field) work only for a spatially flat ($k = 0$) FRW model. Recently, Cimento et al [2000] showed that a combination of dissipative effects such as a bulk viscous stress and a quintessence scalar field gives an accelerated expansion for an open universe ($k = -1$) as well. This model also provides a solution for the 'coincidence problem' as the ratio of the density parameters corresponding to the normal matter and the quintessence field asymptotically approaches a constant value. Bertolami and Martins [2000] obtained an accelerated expansion for the universe in a modified Brans-Dicke (BD) theory by introducing a potential which is a function of the Brans-Dicke scalar field itself. Banerjee and Pavon [2001] have shown that it is possible to have an accelerated universe with BD theory in Friedmann model without any matter.

1.2.3 Chaplygin Gas

In recent years a lot of other models, other than cosmological constant and quintessence scalar fields, have also been proved to provide plausible explanation for dark energy. One of the most popular among these models is Chaplygin gas. Chaplygin Gas unifies dark matter and dark energy under same EOS given by,

$$p = -A/\rho \tag{1.45}$$

where A is a positive constant.

Chaplygin [1904] introduced this EOS to calculate the lifting force on a wing of an airplane in aerodynamics. Later Tsien [1939] and Karman [1941] proposed the same model. Also Stanyukovich [1960] showed that the same EOS can describe certain deformable solids. Jackiw [2000] showed that this is the only gas to admit a supersymmetric generalization. This invokes interest in string theory as well since in a D-brane configuration in the D+2 Nambu-Goto action, the employment of the light-cone parameterization

leads to the action of a newtonian fluid with the EOS (1.45), whose symmetries are the same as the Poincaré group [Colistete Jr., 2002] and also is linked with Born-Infeld model as both have the same D-brane ancestor [Jackiw, 2000] [the parametrization invariant Nambu-Goto D-brane action in a (D + 1, 1) spacetime leads, in the light-cone parametrization, to the Galileo-invariant Chaplygin gas in a (D, 1) spacetime and to the Poincaré-invariant Born-Infeld action in a (D, 1) spacetime]. Thus there exists a mapping between these two systems and their solutions. Thus the Born-Infeld Lagrangian density

$$\mathcal{L}_{BI} = -\sqrt{A}\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\,\theta_{,\nu}} \tag{1.46}$$

gives rise to the EOS (1.45).

As seen before, the metric of a homogeneous and isotropic universe in FRW model (without the Λ-term) is given by equation (1.7). The Einstein field equations are (choosing $8\pi G = c = 1$) given by equations (1.12) and (1.13). The energy conservation equation ($T^{\nu}_{\mu;\nu} = 0$) is given by equation (1.20).

The EOS (1.45) together with these equations give,

$$\rho = \sqrt{A + \frac{B}{a^6}} \tag{1.47}$$

where B is an arbitrary integration constant.

Thus for small values of a, $\rho \sim \frac{\sqrt{B}}{a^3}$ and $p \sim -\frac{A}{\sqrt{B}}a^3$. which implies a dust like matter for small values of a. Also for large values of a, $\rho \sim \sqrt{A}$ and $p \sim -\sqrt{A}$, which implies an empty Universe with cosmological constant A, i.e., the ΛCDM model.

Kamenshchik etal [2001] showed that Chaplygin gas cosmology can interpolate be-

tween different phases of the Universe, starting from dust dominated phase to a de-Sitter Universe passing through an intermediate phase which is a mixture of a cosmological constant and a stiff matter (given by the EOS $p = \rho$) and thus can explain the evolution of the Universe from a decelerated phase to a stage of cosmic acceleration for a flat or open Universe, i.e., for $k = 0$ or $k = -1$. For a closed Universe with $k = 1$ they obtained a static Einstein Universe with $B = \frac{2}{3\sqrt{3}A}$.

Now to find a homogeneous scalar field $\phi(t)$ and a self-interacting potential $V(\phi)$ corresponding to the Chaplygin gas cosmology, we consider the Lagrangian of the scalar field as,

$$\mathcal{L}_{\phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{1.48}$$

The analogous energy density ρ_ϕ and pressure p_ϕ for the scalar field then read,

$$\rho_{\phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \rho = \sqrt{A + \frac{B}{a^6}} \tag{1.49}$$

and

$$p_{\phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi) = -A/\rho = -\frac{A}{\sqrt{A + \frac{B}{a^6}}} \tag{1.50}$$

Hence for flat universe (i.e., $k = 0$) we have

$$\dot{\phi}^2 = \frac{B}{a^6\sqrt{A + \frac{B}{a^6}}} \tag{1.51}$$

and

$$V(\phi) = \frac{1}{2}\sqrt{A}\left(Cosh\ 3\phi + \frac{1}{Cosh\ 3\phi}\right) \tag{1.52}$$

Fabris etal [2001] showed the density perturbations to this model, but, their unperturbed Newtonian equations cannot reproduce the energy density solution of the Chap-

lygin gas cosmology given by equation (1.47) due to choice of lightcone gauge, whereas, Bilic etal [2002] extending this model to large per-turbations by formulating a Zeldovich-like approximation showed that inhomogeneous Chaplygin gas can combine the roles of dark energy and dark matter.

Later Bento etal [2002] generalized the EOS (1.45) to,

$$p = -A/\rho^{\alpha} \tag{1.53}$$

with $0 \leq \alpha \leq 1$, $A > 0$ and obtained generalized Chaplygin gas. It can be seen that for $\alpha = 1$, the above EOS reduces to the pure Chaplygin gas with EOS (1.45).

This is obtained from the generalized Born-Infeld Lagrangian density given by,

$$\mathcal{L}_{GBI} = -A^{\frac{1}{1+\alpha}} \left[1 - (g^{\mu\nu}\theta_{,\mu}\,\theta_{,\nu}\,)^{\frac{1+\alpha}{2\alpha}}\right]^{\frac{\alpha}{1+\alpha}} \tag{1.54}$$

This EOS together with the Einstein equations (1.12) and (1.13) and the conservation law (1.20), gives

$$\rho = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}} \tag{1.55}$$

where B is an arbitrary positive integration constant.

Again for small values of a, $\rho \sim \frac{B^{\frac{1}{1+\alpha}}}{a^3}$ and $p \sim -\frac{A}{B^{\alpha/(1+\alpha))}}a^{3\alpha}$. which implies a dust like matter for small values of a. Also for large values of a, $\rho \sim A^{1/(1+\alpha)}$ and $p \sim -A^{1/(1+\alpha)}$, which implies an empty Universe with cosmological constant $A^{1/(1+\alpha)}$, i.e., the ΛCDM model.

Bento etal [2002] showed that Generalized Chaplygin gas (GCG) cosmology can also explain the evolution of the Universe from dust dominated phase to a de-Sitter Universe

passing through an intermediate phase which is a mixture of a cosmological constant and a soft EOS (given by the EOS $p = \alpha\rho$).

Now we look at the field theoretic approach of this model. Considering the Lagrangian of the scalar field ϕ with potential $V(\phi)$, the corresponding energy density and pressure will be given by,

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \rho = \left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \tag{1.56}$$

and

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = -A\left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{-\frac{\alpha}{1+\alpha}} \tag{1.57}$$

Thus for flat Universe ($k = 0$), the scalar field and the potential will be given by,

$$\phi = \frac{2}{\sqrt{3}(1+\alpha)}\ \mathrm{S}inh^{-1}\left\{\sqrt{\frac{B}{A}}\frac{1}{a^{\frac{3}{2}(1+\alpha)}}\right\} \tag{1.58}$$

and

$$V(\phi) = \frac{1}{2}A^{\frac{1}{1+\alpha}}\mathrm{C}osh^{\frac{2}{1+\alpha}}\left\{\frac{\sqrt{3}(1+\alpha)}{2}\phi\right\} + \frac{1}{2}A^{\frac{1}{1+\alpha}}\mathrm{C}osh^{-\frac{2\alpha}{1+\alpha}}\left\{\frac{\sqrt{3}(1+\alpha)}{2}\phi\right\} \tag{1.59}$$

which reduces to that corresponding to pure Chaplygin gas model if $\alpha = 1$.

Introducing inhomogeneities, Bento etal [2002] have shown that the model evolves consistently with the observations (specially, CMBR peaks, such as Archeops for the location of the first peak and BOOMERANG for the location of the third peak, supernova and high-redshift observations [Bento etal, 2003]) and that the density contrast in this model is less than the CDM model and even gives a better approximation of the ΛCDM model compared to the pure Chaplygin gas model. Later Makler etal [2003] showed that GCG is consistent with SNIa data for any value of $0 \le \alpha \le 1$, although for $\alpha \sim 0.4$, the

case is most favoured. They also examined that the presence of Cosmological constant should rule out the pure Chaplygin gas as per SNAP data, whereas, presence of pure Chaplygin gas should rule out the possibility of cosmological constant in the Universe.

Later Benaoum [2002] further modified this model and proposed to modified Chaplygin gas (MCG) with EOS,

$$p = A\rho - \frac{B}{\rho^{\alpha}} \quad \text{with} \quad 0 \leq \alpha \leq 1 \tag{1.60}$$

With this EOS the energy density takes a much generalized form,

$$\rho = \left[\frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \tag{1.61}$$

where C is an arbitrary integration constant.
Substituting $A = 0$ and $B = A$ we get back the results of GCG.

Debnath etal [2004] have shown that in this model for small value of scale factor Universe will decelerate and for large values of scale factor Universe will accelerate and the transition occurs when $a = \left[\frac{C(1+A)(1+3A)}{2B}\right]^{\frac{1}{3(1+\alpha)(1+A)}}$. They have also examined that this model can describe the evolution of the Universe from radiation era ($A = \frac{1}{3}$ and ρ is very large) to ΛCDM (ρ is very small constant) model and thus can explain the evolution of the Universe to a larger extent than the pure Chaplygin Gas or GCG models.

Considering a scalar field ϕ with self-interacting potential $V(\phi)$, the corresponding energy density and pressure will be,

$$\rho_{\phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \rho = \left[\frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \tag{1.62}$$

and

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = A\rho - \frac{B}{\rho^\alpha} = A\left[\frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} - B\left[\frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}}\right]^{-\frac{\alpha}{1+\alpha}} \tag{1.63}$$

Hence for flat Universe, the scalar field and the potential will be given by,

$$\phi = \frac{2}{\sqrt{3}(1+\alpha)(1+A)}\ Sinh^{-1}\left\{\sqrt{\frac{C(1+A)}{B}}\frac{1}{a^{\frac{3}{2}(1+\alpha)(1+A)}}\right\} \tag{1.64}$$

and

$$V(\phi) = \frac{1}{2}(1-A)\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} Cosh^{\frac{2}{1+\alpha}}\left\{\frac{\sqrt{3}\sqrt{1+A}(1+\alpha)}{2}\phi\right\}$$

$$+\frac{1}{2}B\left(\frac{B}{1+A}\right)^{-\frac{\alpha}{1+\alpha}} Cosh^{-\frac{2\alpha}{1+\alpha}}\left\{\frac{\sqrt{3}\sqrt{1+A}(1+\alpha)}{2}\phi\right\} \tag{1.65}$$

For small values of the scale factor $a(t)$ and large values of ρ, Debnath etal [2004] obtained two different qualitative nature of the potential for $A = 1$ and $A \neq 1$. For $A = 1$, $V(\phi) \to 0$ as $\rho \to \infty$ and for $A \neq 1$, $V(\phi) \to \infty$ as $\rho \to \infty$. For large values of the scale factor $V(\phi) \to \left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}}$ for both the cases.

Chimento and Lazkoz [2005] studied the large-scale perturbations in this model using a Zeldovich-like approximation and showed that this model evolve from a phase dominated by non-relativistic matter to an asymptotically de Sitter phase and that the intermediate regime is described by the combination of dust and a cosmological constant. They also showed that the inhomogeneities introduced in this model evolve consistently with the observations and the density contrast is less than the CDM model and are more similar to ΛCDM or GCG model. Dao-jun Liu and and Xin-zhou Li [2005] investigated this model using the location of the peak of CMBR spectrum and obtained the range for a non-zero A to be, $-0.35 \lesssim A \lesssim 0.025$.

Recently, Jianbo Lu etal [2008] have shown that according to Akaike Information Criterion (AIC) of model selection, recent observational data support the MCG model as well as other popular models.

1.2.4 Tachyonic Field

Recently rolling tachyon condensate has been studied as a source of dark energy. This is a scalar field of non-standard form motivated by string theory as the negative-mass mode of the open string perturbative spectrum [Calacagni etal, 2006]. For strings attached to Dirichlet D-branes such tachyonic modes reflect instability [Das etal, 2005]. The basic idea is that the usual open string vacuum is unstable but there exist a stable vacuum with zero energy density [Gibbons, 2002]. The unstable vacuum corresponds to rolling tachyon. Sen showed that this tachyonic state is associated with the condensation of electric flux tubes of closed strings described by Dirac-Born-Infeld action. For strings attached to Dirichlet D-branes such tachyonic modes reflect D-brane instability [Das etal, 2005]. Tachyonic field has a potential which rolls down from an unstable maximum to a stable minimum with a stable vacuum expectation value. This is known as tachyon condensate [Das etal, 2005]. Sen [2002] has shown that the energy momentum tensor for rolling tachyon solution in D-branes in bosonic string theory is described by a pressureless gas with non-zero energy density, which is stored in open string field, although there are no open string degrees of freedom around the minimum of tachyonic potential. Thus it represents dust, which can be considered as a candidate for CDM. Also the energy-momentum tensor of tachyon condensate can be split into two parts, one with $\omega = 0$ and the other with $\omega = -1$. This has led a lot of authors to construct cosmological models with tachyonic field as a candidate for dark energy, as dark matter and dark energy thus can be described by a single scalar field. Hence in cosmology rolling of tachyon is analogous to the expansion of the Universe [Gibbons, 2002]. Also Sen [2002] showed that the supersymmetry breaking by tachyon matter can be adjusted since the total energy of the tachyon matter is adjustable and is determined by the initial position

and velocity of tachyon.

Now let us move to the dynamics of the tachyon condensate. The Lagrangian density of tachyon condensate is given by the Born-Infeld action

$$\mathcal{L}_{tach} = -V(T)\sqrt{1+g^{\mu\nu}\partial_\mu T\partial_\nu T}\sqrt{-\det(g_{\mu\nu})} = -V(T)\sqrt{-\det(G_{\mu\nu})} \tag{1.66}$$

where T is the tachyonic field, $V(T)$ is the tachyon potential having a local maximum at the origin and a global minimum at $T = \infty$ [Gibbons, 2003] where the potential vanishes.

The tachyon metric is thus given by,

$$G_{\mu\nu} = g_{\mu\nu} + \partial_\mu T\partial_\nu T \tag{1.67}$$

(Another tachyon model has been proposed with Lagrangian $V(T)\sqrt{g^{\mu\nu}\partial_\mu T\partial_\mu T - 1}$, which has been proved to be more effective as to explore more physical situations than quintessence [Chimento, 2003; Srivastava, 2005].

Thus the stress tensor of the tachyonic field is given in the form of a perfect fluid by,

$$T^\mu_\nu = (\rho + p)u^\mu u_\nu - p\delta^\mu_\nu \tag{1.68}$$

where, $u_\mu = \frac{\partial_\mu T}{\sqrt{\partial^\nu T\partial_\nu T}}$, hence

$$\rho = \frac{V(T)}{\sqrt{1-\partial^\nu T\partial_\nu T}} \tag{1.69}$$

and

$$p = -V(T)\sqrt{1-\partial^\nu T\partial_\nu T} \tag{1.70}$$

which for a homogeneous and time dependent tachyonic field reduce to

$$\rho = \frac{V(T)}{\sqrt{1-\dot{T}^2}} \tag{1.71}$$

and

$$p = -V(T)\sqrt{1-\dot{T}^2} \tag{1.72}$$

Hence,

$$p = -\frac{V^2(T)}{\rho} \tag{1.73}$$

and the EOS parameter reads,

$$\omega = \frac{p}{\rho} = -(1-\dot{T}^2) \tag{1.74}$$

Thus $-1 \leq \omega \leq 0$. Also if $V(T)$ is constant, equation (1.73) reduces to the EOS of pure Chaplygin gas.

As for the strong energy condition $\rho + 3p = \frac{V(T)}{\sqrt{1-\dot{T}^2}}(3\dot{T}^2 - 2) > 0$, i.e. SEC fails if $\mid \dot{T} \mid < \sqrt{\frac{2}{3}}$. As seen from the above equations $\mid \dot{T} \mid < 1$.

The equation of motion is

$$\left(g^{\mu\nu} - \frac{\partial^\mu T \partial^\nu T}{1+(\partial T)^2}\right)\partial_\mu \partial_\nu T = -\frac{V'}{V}\left(1+(\partial T)^2\right) \tag{1.75}$$

Considering the gravitational field generated by tachyon condensate and assuming that the cosmological constant term vanishes in the tachyon ground state the action becomes [Gibbons, 2003],

$$S = \int d^4x \left[\frac{R}{16\pi G}\sqrt{-\det(g_{\mu\nu})} - V(T)\sqrt{-\det(G_{\mu\nu})}\right] \tag{1.76}$$

Using this the Raychaudhuri and Friedmann equations give,

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\left[\frac{V(T)}{\sqrt{1-\dot{T}^2}} - \frac{3}{2}\frac{V(T)\dot{T}^2}{\sqrt{1-\dot{T}^2}}\right] \tag{1.77}$$

and

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{k}{a^2} + \frac{8\pi G}{3}\frac{V(T)}{\sqrt{1-\dot{T}^2}} \tag{1.78}$$

The equation of motion reads,

$$\ddot{T} = -(1-\dot{T}^2)\left[\frac{V'(T)}{V(T)} + 3\dot{T}\frac{\dot{a}}{a}\right] \tag{1.79}$$

Also the conservation equation becomes,

$$\dot{\rho} + 3H\rho\dot{T}^2 = 0 \tag{1.80}$$

These equations show that tachyon field rolls down hill with an accelerated motion and the universe expands [Gibbons, 2002]. Raychaudhuri equation shows that initially for small T, i. e., when $\mid T \mid < \sqrt{\frac{2}{3}}$, $\ddot{a} > 0$, i.e., the Universe accelerates and starts decelerating once $\mid T \mid > \sqrt{\frac{2}{3}}$. For flat space-time, i. e., for $k = 0$, $a(t)$ approaches a constant value, for $k = -1$, $a \to t$ and for $k = 1$ re-collapse will take place [Gibbons, 2002].

Tachyonic field can be treated as dark energy or dark matter depending on the form of the potential associated with it. Tachyon potential is usually assumed to be exponentially decaying or inversely quadratic. However, Copeland etal [2005] and Calacagni etal [2006] have carried out the analysis for a wide range of potentials, as given below, although more or less all these models face the problem of fine tuning or are constrained by observational data.

1) $V = V_0 T^{-n}$: For $n < 0$, the model shows instability; for $0 < n < 2$, there is s stable late time attractor solution and V_0 does not need to be fine tuned as super-Planckian problem does not affect this model; for $n = 2$, one gets a power law solution of the form $a = t^m$ and V_0 needs to be fine tuned in order to get present day acceleration, and hence is not a good candidate for dark energy; for $n > 2$, the model has a dust attractor.

2) $V = V_0 e^{1/\alpha T}$, $\alpha > 0$: This model gives an asymptotic de-Sitter solution with V_0 representing the effective cosmological constant.

3) $V = V_0 e^{\alpha^2 T^2}$, $\alpha > 0$: This model gives an oscillating field around the origin with V_0 being the effective cosmological constant.

4) $V = V_0 e^{-\alpha T}$, $\alpha > 0$: This model has a stable dust attractor after a period of acceleration. Also this is large-field approximation of $V = V_0/\cosh(\alpha T)$.

5) $V = V_0 e^{-\alpha^2 T^2}$, $\alpha > 0$: This model is similar to model 4.

Bagla etal [2003] studied the effects of homogeneous tachyon matter in the background of non-relativistic matter and radiation, choosing the inverse square potential and the exponential potential for the tachyonic field and showed that for both these models the density parameter for matter and the tachyons are comparable even in the matter dominated phase. For the exponential potential, we get a phase where $a \propto t^{\frac{2}{3}}$ as $t \to \infty$ preceded by an accelerated expansion and thus eliminating the event horizon present in ΛCDM model. They also carried a supernova Ia data analysis and showed that both the potentials present models where the Universe accelerates at low redshifts and are also consistent with requirements of structure formation.

To study the tachyon driven cosmology Sen [2003] has considered the effective action, taking into account the cosmological constant term, to be,

$$S = -\frac{1}{16\pi G}\int d^4x \left[-\sqrt{-\det(g)}\; R + V(T)\sqrt{-\det(g_{\mu\nu} + \partial_\mu T \partial_\nu T)} + \Lambda\sqrt{-\det(g)}\right] \tag{1.81}$$

with $V(T) = V_0/\cosh(T/\sqrt{2})$ and $T = T(x^0)$.

Implementing the FRW line element the Einstein's field equations and the equations of motion of T are,

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\left[\Lambda + \frac{V(T)}{\sqrt{1-\dot{T}^2}} - \frac{3}{2}\frac{V(T)\dot{T}^2}{\sqrt{1-\dot{T}^2}}\right] \tag{1.82}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{k}{a^2} + \frac{8\pi G}{3}\left[\Lambda + \frac{V(T)}{\sqrt{1-\dot{T}^2}}\right] \tag{1.83}$$

and

$$\ddot{T} = -(1-\dot{T}^2)\left[\frac{V'(T)}{V(T)} + 3\dot{T}\frac{\dot{a}}{a}\right] \tag{1.84}$$

with initial conditions

$$\dot{T} = 0, \qquad \dot{a} = 0, \qquad T = T_0 \quad \text{at} \quad x^0 = 0 \tag{1.85}$$

Sen has used time reversal symmetry and concluded that if Λ is comparatively small rather negligible the universe begins with a big bang and ends in a big crunch. Whereas, in presence of a bulk cosmological constant Universe expands without any singularity for some special range of initial conditions on the tachyon.

Although a lot of shortcomings have been pointed out by many authors [Linde etal, 2002; Shiu etal, 2003] regarding the fine tuning of the model, Gibbons [2003] has stressed on the possibility that the tachyon was important in a pre-inflationary *Open-String Era*

preceding our present *Closed String Era*.

1.2.5 Inhomogeneous EOS

The disturbing features of ΛCDM model discussed above motivate the search for alternatives for standard ΛCDM model, thus causing imhomogeity to be introduced in the EOS so as to account for the present day acceleration. A lot of imhomogeneous models are being studied recently for this purpose.

Garfinkle [2006] has shown that an inhomogeneous but spherically symmetric cosmological model can account for the cosmic acceleration without any dark energy. His model fits the supernova data like the standard FRW model with Λ.

Capozzeillo etal [2006] have investigated the effects of viscosity terms depending on the Hubble parameter and its derivatives in the dark energy EOS. For this purpose they considered two EOS, given by,

$$p = -\rho - A\rho^{\alpha} - BH^{2\beta}$$

and

$$f(p,\ \rho,\ H) = 0$$

where, $A,\ B,\ \alpha,\ \beta$ are constant and f is a function of its arguments.

They present the likelihood analysis to show that both models fit the data given by SNeIa and radio galaxies and predict values of the deceleration parameter and the age of the Universe almost correctly. Nojiri etal [2005] considered a Hubble parameter de-

pendent EOS to construct a late time Universe with $\omega = -1$ crossing. Stefancic [2005] also considered a class of these models to investigate the singularities of the Universe.

Brevik etal [2004] used Cardy-Verlinde formula in a FRW Universe filled with dark energy and obtained the same resualts as modified gravity, which is a gravitational alternative for dark energy. Elizalde etal [2005] have also considered decaying vacuum cosmology and holographic dark energy models motivated by vacuum fluctuations and AdS/CFTlike holographic considerations respectively and have shown that there is no need to introduce exotic matter explicitly, as these models violate the basic energy conditions.

1.3 Brans-Dicke Cosmology

Brans-Dicke (BD) theory has been proved to be very effective regarding the recent study of cosmic acceleration. This theory has very effectively solved the problems of inflation and the early and the late time behaviour of the Universe. The starting point of Brans-Dicke Theory [Brans et al, 1961] is Mach's Principle, that the phenomenon of inertia ought to arise from acceleration w.r.t. the general mass distribution of the Universe. Therefore gravitational acceleration should be used to measure the absolute scale of the elementary particle masses, as they are not constants, rather, represents the particles' interaction with some cosmic field [Weinberg, Gravitation and Cosmology]. Thus the Gravitational Constant G is related to the mass distribution in an expanding Universe by the relation $\frac{GM}{Rc^2} \sim 1$, where R is the radius of the Universe and M is the mass of the Universe, or rather,

$$G^{-1} \sim \sum_i (m_i/r_i c^2) \tag{1.86}$$

where the sum is over all the matter that contribute to the inertial reaction, since both nearby and distant matter should contribute to the inertial reaction. Now if G is to vary

it should be a function of some scalar field variable. Thus if ϕ represents the scalar field coupled to the mass density of the Universe, G should be related to ϕ in some manner. Since a wave equation for ϕ with a scalar matter density as source gives an equation same as equation (1.86), a suitable relation between G and ϕ could be given by $\phi \simeq \frac{1}{G}$. Brans and Dicke proposed a theory in which the correct field equations for gravitation are obtained by replacing G with $\frac{1}{\phi}$. Thus Brans-Dicke Theory is a generalization of the theory of general relativity. Here gravitation effects are described by a scalar field in Riemannian manifold, thus expressing the gravitational effects as both geometrical and due to scalar interactions. They generalize the usual variational principle of general relativity to obtain equations of motion of matter and non-gravitational fields using Einstein field equations, by

$$\delta \int [\phi R + \frac{16\pi}{c^4} L - \omega(\phi_{,i}\, \phi^{ij}/\phi)] = 0 \tag{1.87}$$

where R is the scalar curvature and L is the Lagrangian density of matter including all non-gravitational fields (and not of ϕ) and ω is a dimensionless constant.

Now the conservation laws give,

$$T^{ij}_{;j} = 0 \tag{1.88}$$

where T^{ij} is the energy momentum tensor of matter (excluding ϕ).

The wave equation for ϕ is given by (varying ϕ and $\phi_{,i}$ in equation (1.87))

$$2\omega\phi^{-1}\Box\phi - (\omega/\phi^2)\phi^{,i}\phi_{,i} + R = 0 \tag{1.89}$$

where the generally covariant D'Alembertian $\Box$ is defined to be the covariant divergence of $\phi^{,i}$,

$$\Box\phi = \phi^{;i}_{;i} = (-g)^{-\frac{1}{2}}[(-g)^{-\frac{1}{2}}\phi^{,i}]_{,i} \tag{1.90}$$

Varying the components of the metric tensor and the first derivatives in equation (1.87) the field equations for the metric field are obtained as,

$$R_{ij} - \frac{1}{2}g_{ij}R = (8\pi\phi^{-1}/c^4)T_{ij} + (\omega/\phi^2)(\phi_{,i}\,\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\,\phi^{,k}) + \phi^{-1}(\phi_{,i;j} - g_{ij}\Box\phi) \tag{1.91}$$

which while contracted gives,

$$-R = (8\pi\phi^{-1}/c^4)T - (\omega/\phi^2)\phi_{,k}\,\phi^{,k} - 3\phi^{-1}\Box\phi \tag{1.92}$$

Combining this equation with equation (1.89), the wave equation for ϕ is given by,

$$\Box\phi = \frac{8\pi}{(3+2\omega)c^4}T \tag{1.93}$$

with $ds^2 = g_{ij}dx^i dx^j$, $g_{00} < 0$.

Now for a perfect fluid the energy-momentum tensor given by equation (1.42). Thus we have,

$$T = 3p - \rho \tag{1.94}$$

where, ρ and p are respectively energy density and pressure of the fluid.

Now to apply the Brans-Dicke theory to cosmology, we consider the Universe to be homogeneous and isotropic.

The Robertson-Walker form of the metric is given by equation (1.7).

We write the gravitational field equations (1.91) as,

$$R_{ij} = -\frac{8\pi}{\phi\ c^4}\left[T_{ij} - \left(\frac{1+\omega}{3+2\omega}\right) g_{ij}\ T^{\mu}_{\mu}\right] - \frac{\omega}{\phi^2}\phi_{;i}\phi_{;j} - \frac{1}{\phi}\phi_{;i;j} \tag{1.95}$$

The time-time component of equation (1.95) gives,

$$3\frac{\ddot{a}}{a} = -\frac{8\pi}{(3+2\omega)\phi c^4}\{(2+\omega)\rho + 3(1+\omega)p\} - \omega\frac{\dot{\phi}^2}{\phi^2} - \frac{\ddot{\phi}}{\phi} \tag{1.96}$$

and the space-space component is,

$$-\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a^2} - 2\frac{k}{a^2} = -\frac{8\pi}{(3+2\omega)\phi}\{(1+\omega)\rho - \omega p\} + \frac{\dot{\phi}}{\phi}\frac{\dot{a}}{a} \tag{1.97}$$

and the time-space gives,

$$0 = 0 \tag{1.98}$$

Also the field equation for ϕ, i.e., equation (1.93) gives,

$$\frac{d}{dt}(\dot{\phi}a^3) = \frac{8\pi}{(3+2\omega)c^4}(\rho - 3p)a^3 \tag{1.99}$$

and the conservation law is given by equation (1.20).

Equation (1.99), on simplification gives,

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = (\rho - 3p)\frac{8\pi}{(3+2\omega)c^4} \tag{1.100}$$

Using equations (1.96), (1.97) and (1.100), we get,

$$3\frac{\dot{a}^2 + k}{a^2} = \frac{8\pi}{c^4}\frac{\rho}{\phi} - 3\frac{\dot{\phi}}{\phi}\frac{\dot{a}}{a} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} \tag{1.101}$$

and

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} = -\frac{8\pi}{c^4}\frac{p}{\phi} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{\phi}}{\phi}\frac{\dot{a}}{a} - \frac{\ddot{\phi}}{\phi} \tag{1.102}$$

Equations (1.99), (1.100), (1.101), (1.102) are the fundamental equations of Brans-Dicke Cosmology. In these equations the Brans-Dicke parameter 'ω' is kept as a constant.

Solar system experiments impose a limit on the value of ω, i.e., $\mid \omega \mid \geq 500$, although ω is seen to have a low negative value in order to solve the cosmic acceleration and coincidence problem. Also a constant negative ω fails to give a consistent radiation model which explains the primordial nucleosysnthesis. Banerjee and Pavon [2001] have shown that this can be solved by a varying ω theory where ω is considered to be a function of the scalar field ϕ [Nordtvedt, 1970; Bergmann, 1968; Wagoner, 1970]. Therefore, the action integral for this general class of scalar-tensor gravitational theory is,

$$A = \int \left[16\pi L + \phi R + \frac{\omega(\phi)}{\phi}\phi_{;\mu}\phi^{;\mu}\right]\sqrt{-g}\ d^4x \tag{1.103}$$

Thus the field equations for the tensor and scalar fields become,

$$\Box\phi = \frac{8\pi}{(3+2\omega)c^4}T - \frac{\omega'}{3+2\omega}\phi_{,\mu}\,\phi^{,\mu} \tag{1.104}$$

together with equation (1.91), where, $\omega' = \frac{d\omega}{d\phi}$.

Thus these equations can be combined to get,

$$R_{ij} = -\frac{8\pi}{\phi c^4}\left[T_{ij} - \frac{1+\omega}{3+2\omega}g_{ij}T^{\mu}_{\mu}\right] - \frac{\omega}{\phi^2}\phi_{;i}\phi_{;j} - \frac{1}{\phi}\phi_{;i}\phi_{;j} + \frac{\omega'}{2\phi(3+2\omega)}\phi_{;i}\phi^{;i}g_{ij} \tag{1.105}$$

This modifies equation (1.100) by,

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = (\rho - 3p)\frac{8\pi}{(3+2\omega)c^4} - \frac{\dot{\omega}\dot{\phi}}{3+2\omega} \tag{1.106}$$

Now the effect of $\omega' \neq 0$ will be in $g_{\mu\nu}$ only in the non-linear order in the mass source strength. Hence, gravitational fields in linear mass are identical with the results of Brans-Dicke theory with $\omega =$ constant. Banerjee and Pavon [2001] have shown that this varying ω-theory can potentially solve the quintessence problem and give rise to a non-decelerating radiation model also. On the other hand, Bertolami and Martins [2000] obtained an accelerated expansion of the Universe in a further modified form of Brans-Dicke Theory by introducing a potential which is a function of the scalar field. This self-interacting Brans-Dicke Theory is described by the action,

$$S = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}\phi^{,\alpha}\phi_{,\alpha} - V(\phi) + 16\pi\mathcal{L}_m\right] \tag{1.107}$$

where, $V(\phi)$ is the self-interacting potential for the Brans-Dicke scalar field ϕ. Thus the field equations are obtained as,

$$G_{\mu\nu} = \frac{\omega}{\phi^2}\left[\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}\right] + \frac{1}{\phi}\left[\phi_{,\mu;\nu} - g_{\mu\nu}\Box\phi\right] - \frac{V(\phi)}{2\phi}g_{\mu\nu} + \frac{T_{\mu\nu}}{\phi}\frac{8\pi}{c^4} \tag{1.108}$$

$$\Box\phi = \frac{8\pi}{(3+2\omega)c^4}T - \frac{1}{3+2\omega}\left[2V(\phi) - \phi\frac{dV(\phi)}{d\phi}\right] \tag{1.109}$$

These field equations under Friedmann-Robertson-Walker geometry modifies equations (1.100), (1.101) and (1.102) as,

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = (\rho - 3p)\frac{8\pi}{(3+2\omega)c^4} + \frac{1}{3+2\omega}\left[2V(\phi) - \phi\frac{dV(\phi)}{d\phi}\right] \tag{1.110}$$

$$3\frac{\dot{a}^2 + k}{a^2} = \frac{8\pi}{c^4}\frac{\rho}{\phi} - 3\frac{\dot{\phi}}{\phi}\frac{\dot{a}}{a} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{V}{2\phi} \tag{1.111}$$

and

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} = -\frac{8\pi}{c^4}\frac{p}{\phi} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{\phi}}{\phi}\frac{\dot{a}}{a} - \frac{\ddot{\phi}}{\phi} + \frac{V}{2\phi} \tag{1.112}$$

Bertolami and Martins have obtained the solution for accelerated expansion with a quadratic self-coupling potential ($V(\phi) \sim \phi^2$) and a negative coupling constant ω, although they have not considered the positive energy conditions for the matter and scalar field. Amendola [1999] has shown that coupled quintessence models are conformally equivalent to Brans-Dicke Lagrangians with power-law potential given by, $V(\phi) \sim \phi^n$. Sen etal [2001] have studied the late time acceleration in the context of Brans Dicke (BD) theory with potential $V(\phi) = \lambda\phi^4 - \mu^2(t)\phi^2$, and showed that a fluid with dissipative pressure can drive this late time acceleration for a simple power law expansion of the universe, whereas, a perfect fluid cannot support this acceleration. Later Chiba [2003] extended the gravity theories and obtained a BD theory with potential for BD scalar field, but found that this is not compatible with solar system experiments if the field is very light.

1.4 Statefinder Diagnostics

Over the years a lot of models have proved to be viable candidates of Dark Energy, thus leading to the problem of discriminating between these models. For this purpose Sahni etal [2003] proposed a new geometric diagnosis (dimensionless) to characterize the properties of dark energy in a model independent manner. They introduced a pair of parameters called *statefinder parameters* depending on the scale factor and its derivatives, defined by,

$$r = \frac{\dddot{a}}{aH^3}, \qquad s = \frac{r-1}{3(q-1/2)} \tag{1.113}$$

where $q = -\frac{a\ddot{a}}{\dot{a}^2}$ is the deceleration parameter. The parameter r forms the next step in the hierarchy of geometrical cosmological parameters after H and q. In fact trajectories in the $\{s, r\}$ plane corresponding to different cosmological models demonstrate qualitatively different behaviour, for example ΛCDM model correspond to the fixed point $s = 0,\ r = 1$..

For spatially flat space-time ($k = 0$), considering the Universe to be consisted of non-relativistic matter Ω_m, i.e., CDM and baryons, and dark energy $\Omega_x = 1 - \Omega_m$, the statefinder pair $\{r, s\}$ takes the form, [Sahni etal, 2003]

$$r = 1 + \frac{9}{2}\Omega_x \omega(1+\omega) - \frac{3}{2}\Omega_x \frac{\dot{\omega}}{H} \tag{1.114}$$

and

$$s = 1 + \omega - \frac{1}{3}\frac{\dot{\omega}}{\omega H} \tag{1.115}$$

Thus for ΛCDM model with a non-zero Λ ($\omega = -1$), $r = 1$ and $s = 0$.

If ω is constant these parameters reduce to

$$r = 1 + \frac{9}{2}\Omega_x \omega(1+\omega), \qquad s = 1 + \omega \tag{1.116}$$

Degeneracy occurs when $\omega = -1/3$ or $\omega = -2/3$, as $r \to 1$ at earlier stage and $r \to 0$ at later age, with $r \simeq 0.3$ for present age ($\Omega_x \simeq 0.7$) at the present time.

For a quintessence scalar field these parameters take the forms

$$r = 1 + \frac{12\pi G\dot{\phi}^2}{H^2} + \frac{8\pi G\dot{V}}{H^3} \tag{1.117}$$

and

$$s = \frac{2(\dot{\phi}^2 + 2\dot{V}/3H)}{\dot{\phi}^2 - 2V} \tag{1.118}$$

For pure ($p = -B/\rho, (B > 0)$) and generalized ($p = -B/\rho^\alpha, 0 \le \alpha \le 1$) Chaplygin Gas, we get respectively, [Gorini etal, 2002]

$$r = 1 - \frac{9}{2}s(1+s) \tag{1.119}$$

and

$$r = 1 - \frac{9}{2}s(\alpha+s)/\alpha \tag{1.120}$$

For modified Chaplygin gas ($p = A\rho - \frac{B}{\rho^\alpha}$ with $0 \leq \alpha \leq 1$), these parameters have rather an implicit form given by, [Debnath etal, 2004]

$$18(r-1)s^2 + 18\alpha s(r-1) + 4\alpha(r-1)^2 = 9sA(1+\alpha)(2r+9s-2) \tag{1.121}$$

In general, for one fluid model, these $\{r, s\}$ can be written as

$$r = 1 + \frac{9}{2}\left(1 + \frac{p}{\rho}\right)\frac{\partial p}{\partial \rho} \tag{1.122}$$

and

$$s = \left(1 + \frac{\rho}{p}\right)\frac{\partial p}{\partial \rho} \tag{1.123}$$

Gorini etal have shown that for pure Chaplygin gas s varies in the interval $[-1,\ 0]$ and r first increases from r = 1 to its maximum value and then decreases to the ΛCDM fixed point $s = 0,\ r = 1$. For generalized Chaplygin gas, the model becomes identical with the standard ΛCDM model for small values of α from statefinder viewpoint. Debnath etal have shown that in case of modified Chaplygin gas the Universe can be described from radiation era to ΛCDM with statefinder diagnosis.

Alam etal [2003] have shown that s is positive for quintessence models, but negative for the Chaplygin gas models, whereas, r is < 1 or > 1 for quintessence or Chaplygin gas.

Later Shao and Gui [2007] carried out the statefinder diagnosis on tachyonic field and showed that the tachyon model can be distinguished from the other dark energy models by statefinder diagnosis as the evolving trajectories of the attractor solutions lie in the total region although they pass through the LCDM fixed point.

Using SNAP data Alam etal have also demonstrated that the Statefinder can distinguish a cosmological constant ($\omega = -1$) from quintessence models with $\omega > -0.9$ and Chaplygin gas models with $\kappa \leq 15$ ($\kappa = \frac{\Omega_m}{1-\Omega_m}$) at the 3σ level if the value of Ω_m is known. Even if the value of Ω_m is known to approximately 20% accuracy statefinder diagnosis rule out quintessence with $\omega > -0.85$ and the Chaplygin gas with $\kappa \leq 7$ again at 3σ level. They have shown that the statefinder diagnosis can differentiate between various dark energy models at moderately high redshifts of $z \lesssim 10$.

Chapter 2

Cosmological Dynamics of MCG in presence of Barotropic Fluid

2.1 Prelude

Chaplygin Gas Cosmology has been studied by a lot of authors to get a plausible model of Dark Energy [Sahni etal, 2000; Peebles etal, 2003; Padmanabhan, 2003]. Pure Chaplygin Gas [Kamenshchik etal, 2001] with EOS $p = -B/\rho, (B > 0)$ behaves as pressureless fluid for small values of the scale factor and as a cosmological constant for large values of the scale factor which tends to accelerate the expansion. Subsequently the above equation was generalized (GCG) to the form $p = -B/\rho^{\alpha}, 0 \leq \alpha \leq 1$ [Gorini etal, 2003; Alam etal, 2003; Bento etal, 2002] and recently it was modified to the form $p = A\rho - B/\rho^{\alpha}, (A > 0)$ [Benaoum, 2002; Debnath etal, 2004], which is known as Modified Chaplygin Gas (MCG). Further Gorini etal considered a two fluid model consisting of Chaplygin Gas and a dust component and showed that for some particular values of the parameters this model can solve the cosmic coincidence problem.

Over the time the statefinder diagnostics have become very popular to discriminate between various dark energy models. In fact trajectories in the $\{r, s\}$ plane corresponding to different cosmological models demonstrate qualitatively different behaviour. Debnath etal [2004] carried out the statefinder diagnostics for MCG and showed that this model shows a radiation era ($A = 1/3$) at one extreme and a ΛCDM model at the other extreme. Gorini etal [2003] showed that their two fluid model is indistinguishable from

ΛCDM model for some values of the parameters from statefinder point of view and thus is quite different from the pure Chaplygin Gas.

In this chapter, we have generalized the model proposed by Gorini etal [2003]. We have considered a two fluid model consisting of modified Chaplygin gas and barotropic fluid. We have analysed this model to study the cosmological evolution of the Universe. Also we have carried out the statefinder analysis to describe the different phases of the evolution and the significance of this model in comparison with the one fluid model of MCG.

2.2 Field Equations and Solutions

The metric of a homogeneous and isotropic universe in FRW model is given by equation (1.7). The Einstein field equations are (1.12) and (1.13) and the energy conservation equation is (1.20). For modified Chaplygin gas, the energy density is given by equation (1.61).

Here we consider two fluid cosmological model which besides a modified Chaplygin's component, with EOS (1.60) contains also a barotropic fluid component with equation of state $p_1 = \gamma\rho_1$. Normally for accelerating universe γ satisfies $-1 \leq \gamma \leq 1$. But observations state that γ satisfies $-1.6 \leq \gamma \leq 1$ i.e., $\gamma < -1$ corresponds to phantom model. For these two component fluids ρ and p should be replaced by $\rho + \rho_1$ and $p + p_1$ respectively. Here we have assumed the two fluid are separately conserved. For Chaplygin gas, the density has the expression given in equation (1.62) and for another fluid, the conservation equation gives the expression for density as

$$\rho_1 = \frac{d}{a^{3(1+\gamma)}} \tag{2.1}$$

where d is an integration constant.

2.3 Field Theoretical Approach

We can describe this two fluid cosmological model from the field theoretical point of view by introducing a scalar field ϕ and a self-interacting potential $V(\phi)$ with the effective Lagrangian is given by (1.48). The analogous energy density ρ_ϕ and pressure p_ϕ corresponding scalar field ϕ having a self-interacting potential $V(\phi)$ are the following:

$$\rho_\phi = \frac{1}{2}\,\dot{\phi}^2 + V(\phi) = \rho + \rho_1 = \left[\frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} + \frac{d}{a^{3(1+\gamma)}} \tag{2.2}$$

and

$$p_\phi = \frac{1}{2}\,\dot{\phi}^2 - V(\phi) = p + p_1 = A\rho - \frac{B}{\rho^\alpha} + \gamma\rho_1 = A\left[\frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} - B\left[\frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}}\right]^{-\frac{\alpha}{1+\alpha}} + \frac{\gamma\, d}{a^{3(1+\gamma)}} \tag{2.3}$$

For flat Universe ($k=0$) and by the choice $\gamma = A$, we have the expression for ϕ and $V(\phi)$:

$$\phi = -\frac{1}{\sqrt{3(1+A)}\,(1+\alpha)} \int \left[\frac{d + c\left(C + \frac{Bz}{1+A}\right)^{-\frac{\alpha}{1+\alpha}}}{d + \left(C + \frac{Bz}{1+A}\right)^{\frac{1}{1+\alpha}}}\right]^{\frac{1}{2}} \frac{dz}{z} \tag{2.4}$$

and

$$V(\phi) = A\left[\frac{B}{1+A} + \frac{C}{z}\right]^{\frac{1}{1+\alpha}} - B\left[\frac{B}{1+A} + \frac{C}{z}\right]^{-\frac{\alpha}{1+\alpha}} + \frac{(1-A)\,d}{z^{\frac{1}{1+A}}} \tag{2.5}$$

where $z = a^{3(1+A)(1+\alpha)}$.

The graphical representation of ϕ against a and $V(\phi)$ against a and ϕ respectively have been shown in figures 2.1 - 2.3 for $A = 1/3$ and $\alpha = 1$. From figure 2.1 we have seen that scalar field ϕ decreases when scale factor $a(t)$ increases for $A = 1/3$. In figure 2.2, we see that potential function $V(\phi)$ sharply decreases from extremely large value

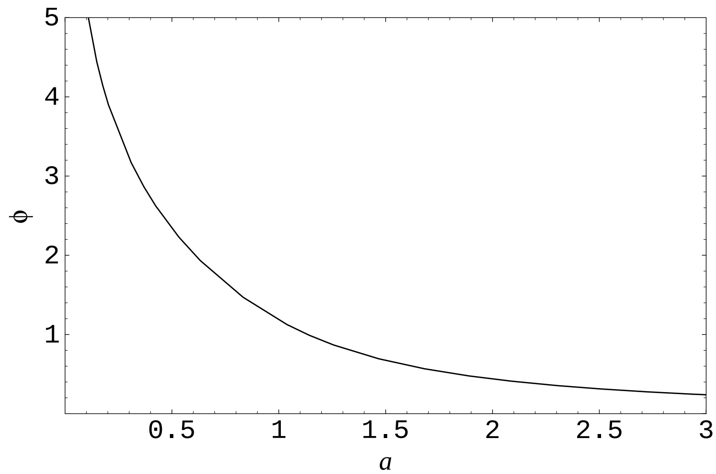

Fig 2.1: Here variation of ϕ has been plotted against a for $A(=\gamma)=1/3$ and $\alpha=1$ (values of other constants: $B=1, C=1, d=1$)

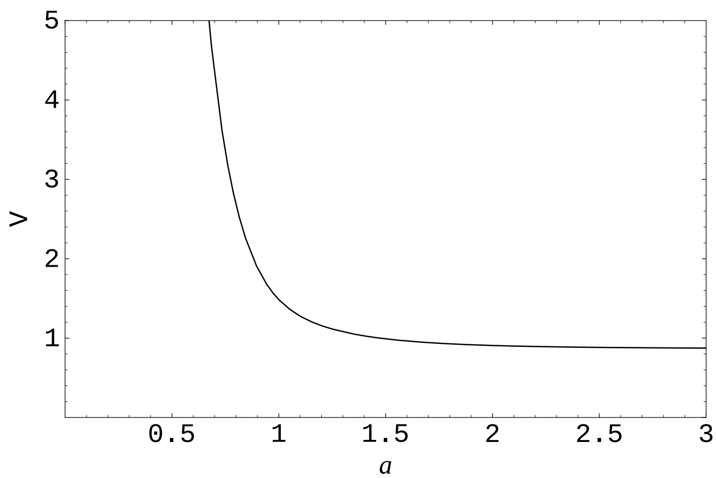

Fig 2.2: Here variation of V has been plotted against a for $A(=\gamma)=1/3$ and $\alpha=1$ (values of other constants: $B=1, C=1, d=1$)

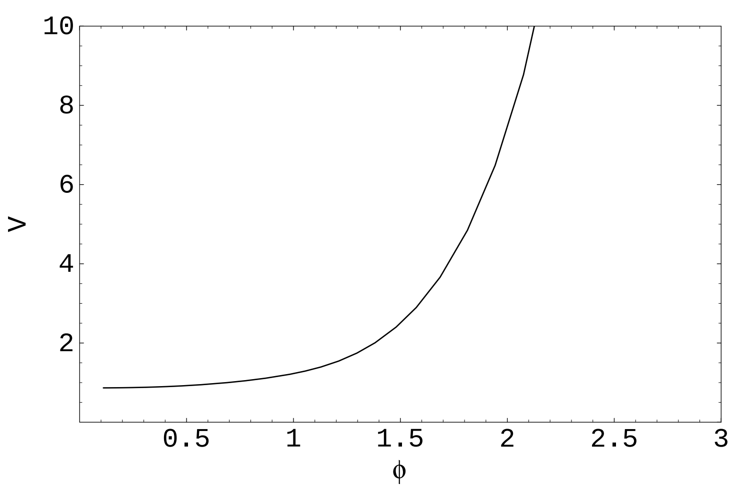

Fig 2.3: Here variation of V has been plotted against ϕ for $A(=\gamma)=1/3$ and $\alpha=1$ (values of other constants: $B=1, C=1, d=1$)

to a fixed value for $A = 1/3$. The potential function $V(\phi)$ increases to infinitely large value when scale factor $a(t)$ increases for $A = 1/3$. So the figures show how $V(\phi)$ varies with ϕ and $a(t)$.

2.4 Statefinder Diagnosis

We have already studied the significance of statefinder diagnosis of the models. Let us now analyse our model using statefinder parameters. The statefinder diagnostic pair has the form given by equation (1.113). For one fluid model, these $\{r, s\}$ can be given by equations (1.122) and (1.123).

For the two component fluids, these equations take the following form:

$$r = 1 + \frac{9}{2(\rho + \rho_1)}\left[\frac{\partial p}{\partial \rho}(\rho + p) + \frac{\partial p_1}{\partial \rho_1}(\rho_1 + p_1)\right] \tag{2.6}$$

and

$$s = \frac{1}{(p + p_1)}\left[\frac{\partial p}{\partial \rho}(\rho + p) + \frac{\partial p_1}{\partial \rho_1}(\rho_1 + p_1)\right] \tag{2.7}$$

The deceleration parameter q has the form:

$$q = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} + \frac{3}{2}\left(\frac{p + p_1}{\rho + \rho_1}\right) \tag{2.8}$$

For modified gas and barotropic equation states, we can set:

$$x = \frac{p}{\rho} = A - \frac{B}{\rho^{\alpha+1}} \tag{2.9}$$

and

$$y = \frac{\rho_1}{\rho} = \frac{\frac{d}{a^{3(1+\gamma)}}}{\left[\frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}} \tag{2.10}$$

Thus equations (2.6) and (2.7) can be written as

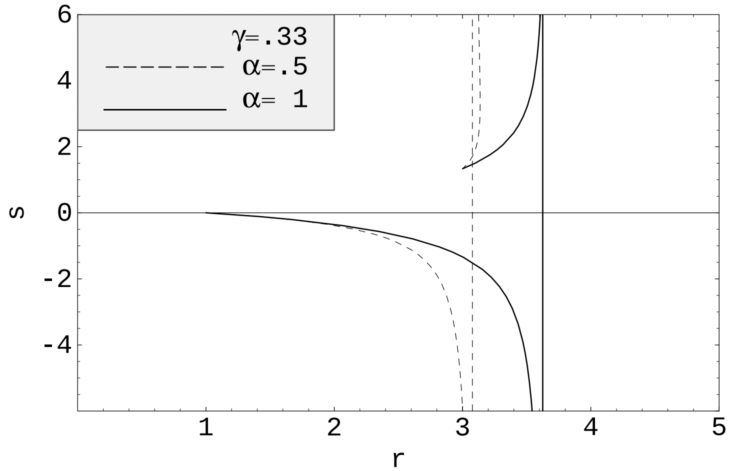

Fig 2.4: Here variation of s has been plotted against r for $\gamma = 1/3$, α (= 0.5, 1)and $A = 1/3$.

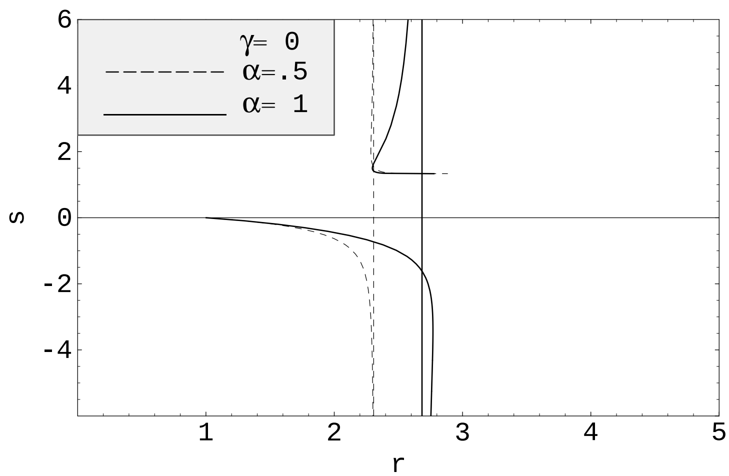

Fig 2.5: Here variation of s has been plotted against r for $\gamma = 0$, α (= 0.5, 1) and $A = 1/3$.

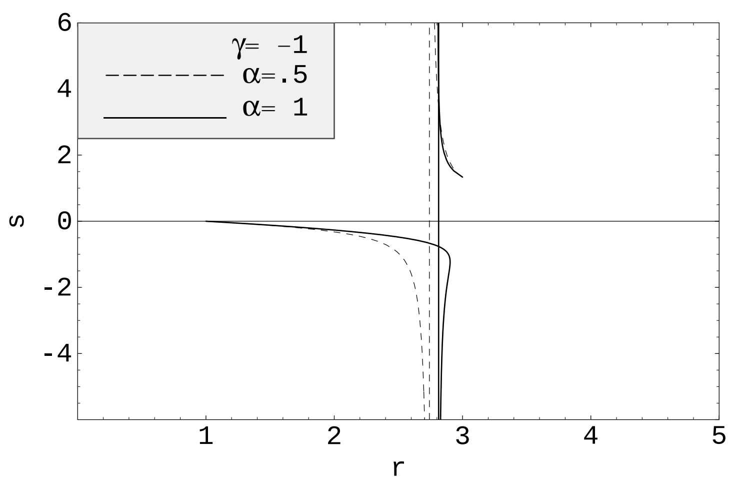

Fig 2.6: Here variation of s has been plotted against r for $\gamma = -1$, α (= 0.5, 1) and $A = 1/3$.

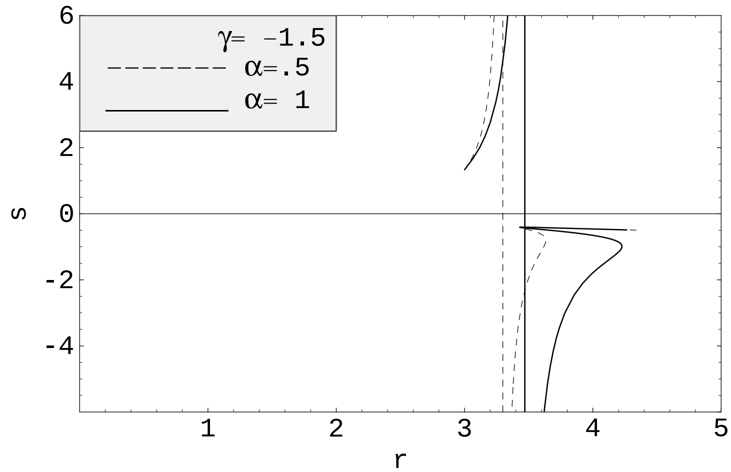

Fig 2.7: Here variation of s has been plotted against r for $\gamma = -1.5$, α (= 0.5, 1)and $A = 1/3$.

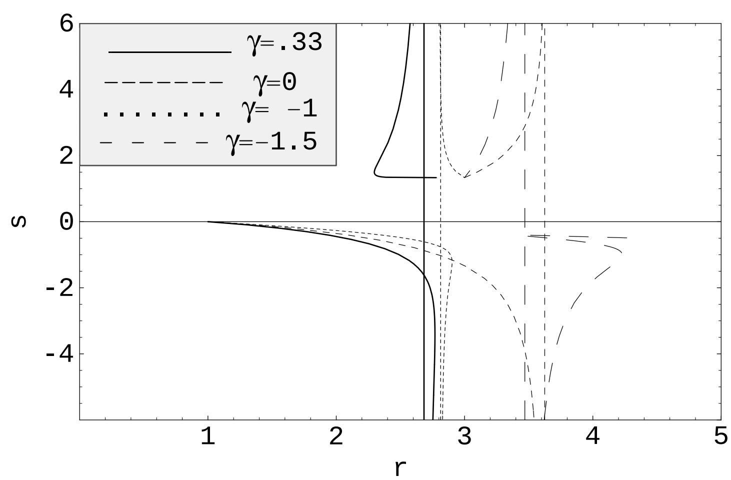

Fig 2.8: This figure shows the variation of s against r for different values of $\gamma = 1/3,\ 0,\ -1,\ -1.5$ and for $\alpha\ (=1)$, $A = 1/3$.

$$r = 1 + \frac{9s}{2}\left(\frac{x+\gamma y}{1+y}\right) \tag{2.11}$$

and

$$s = \frac{(1+x)\{A(1+\alpha) - \alpha x\} + \gamma(1+\gamma)y}{x+\gamma y} \tag{2.12}$$

with

$$y = \left[\frac{d^{(1+\alpha)(1+A)}B^{\gamma-A}(1+\gamma)^{1+\gamma}}{C^{1+\gamma}(1+A)^{1+\gamma}(A-x)^{\gamma-A}}\right]^{\frac{1}{(1+\alpha)(1+A)}} \tag{2.13}$$

From the equations (2.11) and (2.12) we can not write the relationship between r and s in closed form. Thus the relation between the parameters r and s in $\{r, s\}$ plane for different choices of other parameters are plotted in figures 2.4 - 2.8. The figures 2.4 - 2.7 shows the variation of s against r for different values of $\gamma = 1/3,\ 0,\ -1,\ -1.5$ respectively and for $\alpha\ (=\ 0.5,\ 1)$, $A = 1/3$. Fig 2.8 shows the variation of s against r for different values of $\gamma = 1/3,\ 0,\ -1,\ -1.5$ and for $\alpha\ (= 1)$, $A = 1/3$. Thus the figures 2.4 - 2.6 represent the evolution of the universe starting from the radiation era to the ΛCDM model for $\gamma = 1/3,\ 0,\ -1$ and the figure 2.7 represents the evolution of the universe starting from the radiation era to the quiessence model for $\gamma = -1.5$. Thus γ plays an active role for the various stages of the evolution of the universe. If we choose the arbitrary constant d is equal to zero, we recover the model of Modified Chaplygin gas [Debnath etal, 2004]. If A and the barotropic index γ are chosen to be zero, we get back to the results of the works of Gorini etal [2003].

2.5 Discussion

In this chapter, we have analysed a model consisting of modified Chaplygin gas and barotropic fluid. As We have shown that the mixture of these two fluid models is valid from (i) the radiation era to ΛCDM for $-1 \le \gamma \le 1$ and (ii) the radiation era to

quiessence model for $\gamma < -1$. We have carried out the statefinder diagnosis for this model and presented the result graphically. The graphical representation show the validation of this model for different phases of the evolution of the Universe. During the various stages of the evolution we see that γ plays a very important role. It depends on γ whether the barotropic fluid will behave as dark matter or dark energy. For $\gamma = 0$ this fluid is dust, for $\gamma = \frac{1}{3}$, it represents radiation and for $\gamma < 0$ it implies negative pressure, thus determining the nature of the barotropic fluid, whereas MCG unifies dark matter and dark energy under the same umbrella. Also equation (2.10) shows that for $A = \gamma$ at the initial stage with a parameter $\kappa = \frac{d}{C^{1/(1+\alpha)}}$ of order one, the initial energies of MCG and barotropic fluid are of same order of magnitude, which may provide a solution to the cosmic coincidence problem.

Chapter 3

Effect of Dynamical Cosmological Constant in presence of MCG

3.1 Prelude

There are two parameters, the cosmological constant Λ and the gravitational constant G, present in Einstein's field equations. The Newtonian constant of gravitation G plays the role of a coupling constant between geometry and matter in the Einstein's field equations. In an evolving Universe, it appears natural to look at this "constant" as a function of time. Numerous suggestions based on different arguments have been proposed in the past few decades in which G varies with time [Wesson, 1978, 1980]. Dirac [1979] proposed a theory with variable G motivated by the occurrence of large numbers discovered by Weyl, Eddington and Dirac himself.

It is widely believed that the value of Λ was large during the early stages of evolution and strongly influenced its expansion, whereas its present value is incredibly small [Weinberg, 1989; Carroll etal, 1992]. We have already discussed in the introduction that several authors [Freese etal, 1987; Ozer and Taha, 1987; Gasperini, 1987, 1998; Chen and Wu, 1990] have advocated a variable Λ in the framework of Einstein's theory to account for this fact. Λ as a function of time has also been considered in various variable G theories in different contexts [Banerjee etal, 1985; Bertolami, 1986; Abdussattar and Vishwakarma, 1997; Kalligas etal, 1992]. For these variations, the energy-momentum tensor of matter leaves the form of the Einstein's field equations unchanged.

In attempt to modify the General Theory of Relativity, Al-Rawaf and Taha [1996] related the cosmological constant to the Ricci Scalar $\mathcal{R}$. This is written as a built-in-cosmological constant, i.e., $\Lambda \propto \mathcal{R}$. Since the Ricci Scalar contains a term of the form $\frac{\ddot{a}}{a}$, one adopts this variation for Λ. We parameterized this as $\Lambda \propto \frac{\ddot{a}}{a}$ [Arbab, 2003, 2004]. Similarly, we have chosen another two forms for $\Lambda : \Lambda \propto \rho$ and $\Lambda \propto \frac{\dot{a}^2}{a^2}$ [Carvalho, 1992]; where ρ is the energy density.

In this chapter we have considered the Universe to be filled with Modified Gas and the Cosmological Constant Λ to be time-dependent with or without the Gravitational Constant G to be time-dependent. We have considered various phenomenological models for Λ , viz., $\Lambda \propto \rho, \Lambda \propto \frac{\dot{a}^2}{a^2}$ and $\Lambda \propto \frac{\ddot{a}}{a}$. Also we have shown the natures of G and Λ over the total age of the Universe and analysed our models in the viewpoint of satefinder diagnostics.

3.2 Einstein Field Equations with Dynamic Cosmological Constant

We consider the spherically symmetric FRW metric (1.7). The Einstein field equations for a spatially flat Universe (i.e., taking $k = 0$) with a time-dependent cosmological constant $\Lambda(t)$ are given by (choosing $c = 1$),

$$3\frac{\dot{a}^2}{a^2} = 8\pi G\rho + \Lambda(t) \tag{3.1}$$

and

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -8\pi Gp + \Lambda(t) \tag{3.2}$$

where ρ and p are the energy density and isotropic pressure respectively.

Let us choose MCG with EOS given by equation (1.60). Here, we consider the phenomenological models for $\Lambda(t)$ of the forms $\Lambda \propto \rho$, $\Lambda \propto \frac{\dot{a}^2}{a^2}$ and $\Lambda \propto \frac{\ddot{a}}{a}$.

First we will consider G to be constant and try to find out the solutions for density ρ and the scale factor $a(t)$ and hence study the cosmological models in terms of the statefinder parameters r, s. Secondly we will consider G to be variable as well and study the various phases of the Universe represented by the models.

3.3 Models keeping G constant and Λ variable

Taking G to be constant and Λ to be time dependent, the energy conservation equation is,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -\frac{\dot{\Lambda}}{8\pi G} \tag{3.3}$$

3.3.1 Model with $\Lambda \propto \rho$

Here we consider

$$\Lambda = \beta_1 \ \rho \tag{3.4}$$

where β_1 is a constant.

Equation (3.4) together with equations (1.61) and (3.3) yield the solution for ρ to be,

$$\rho = \left(\frac{B}{1+A} + \frac{C}{a^{\frac{24\pi G(1+A)(1+\alpha)}{8\pi G+\beta_1}}} \right)^{\frac{1}{1+\alpha}} \tag{3.5}$$

where C is an arbitrary constant.

Substituting equation (3.4) and (3.5) in equation (3.1), we get the solution for the scale factor $a(t)$ as,

$$a^{f_1 f_2}\sqrt{8\pi G + \beta_1} \ \ {}_2F_1[f_2, f_2, 1+f_2, -\frac{a^{f_1 B}}{C(1+A)}] = 4\sqrt{3}(1+A)G\pi C^{f_2} \ t \tag{3.6}$$

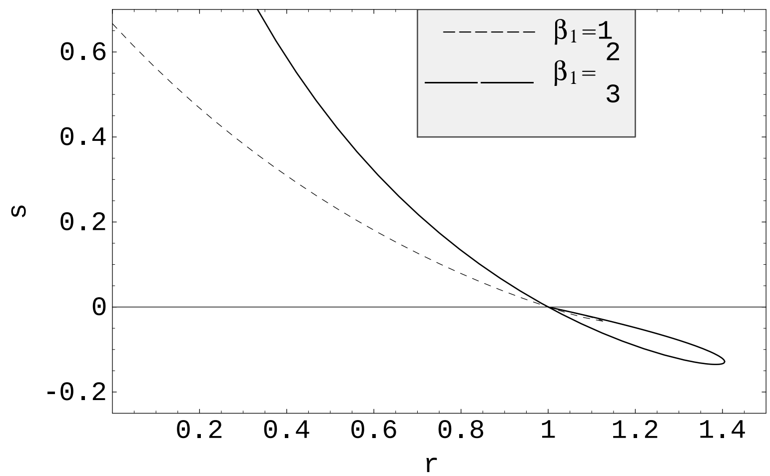

Fig 3.1: This figure shows the variation of s against r for different values of $\beta_1 = 1, \frac{2}{3}$ respectively and for $\alpha = 1$, $A = 1/3$ and $8\pi G = 1$.

where $f_1 = \frac{24(1+A)(1+\alpha)\pi G}{8\pi G+\beta_1}$ and $f_2 = \frac{1}{2(1+\alpha)}$. Hence, for small values of $a(t)$, we have, $\rho \simeq \left(\frac{C}{a^{\frac{24\pi G(1+A)(1+\alpha)}{8\pi G+\beta_1}}}\right)^{\frac{1}{1+\alpha}}$ which is very large and the EOS (1.61) reduces to $p \simeq A\rho$. Again for large values of $a(t)$, we get $\rho \simeq \left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}}$ and $p \simeq -\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}}$, i.e., $p \simeq -\rho$ which coincides with the result obtained for MCG with $\beta_1 = 0$ [Gorini etal, 2003; Alam etal, 2003; Bento etal, 2002].

Using equations (3.1) and (3.3) in statefinder equations (1.113) we get,

$$r = 1 + \frac{36\pi G(1+y)[8\pi G\{A(1+\alpha) - y\alpha\} - \beta_1]}{(8\pi G + \beta_1)^2} \tag{3.7}$$

and

$$s = \frac{8\pi G(1+y)[8\pi G\{A(1+\alpha) - y\alpha\} - \beta_1]}{(8\pi G + \beta_1)(8\pi G y - \beta_1)} \tag{3.8}$$

where $y = \frac{p}{\rho}$ which can be further reduced to a single relation between r and s. Now $q = -\frac{\ddot{a}}{aH^2} = \frac{8\pi G(1+3y)-2\beta_1}{2(8\pi G+\beta_1)}$. Therefore for acceleration $q < 0 \;\Rightarrow\; y < \frac{\beta_1}{12\pi G} - \frac{1}{3}$. Also for the present epoch $q = -\frac{1}{2} \;\Rightarrow\; y = \frac{1}{3}(\frac{3\beta_1}{8\pi G} - 2)$. If we assume that the present Universe is dust filled, we have $y = 0$, i.e., $\beta_1 = \frac{16\pi G}{3}$. Taking $8\pi G = 1$ we get the best fit value to be $\beta_1 = \frac{2}{3}$, which gives $r = 1$ (choosing $A = \frac{1}{3}$, $\alpha = 1$) for the present Universe. That means the dark energy responsible for the the present acceleration is

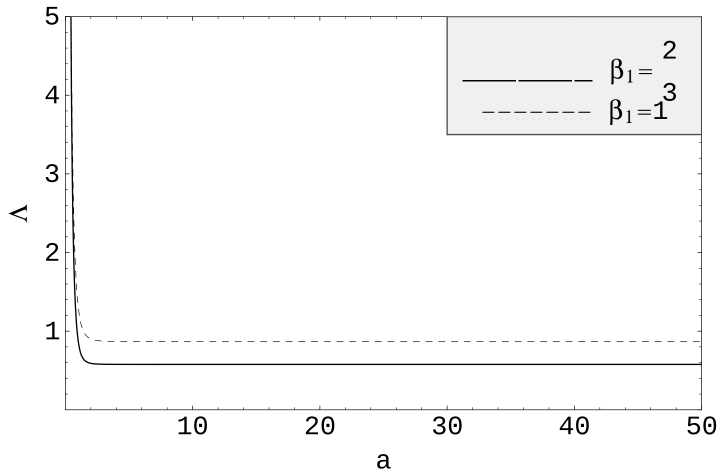

Fig 3.2: Here the variation of Λ has een plotted against $a(t)$ for different values of $\beta_1 = 1, \frac{2}{3}$ respectively and for $\alpha = 1$, $A = 1/3$, $8\pi G = 1$, $B = 1$, $C = 1$.

nothing but Λ. Also $A = 1$, $\alpha = 1$ and $\beta_1 = \frac{2}{3}$ give $r = 2.16$ for the present time. For this case $\beta_1 > 3$ gives non-feasible solutions in the sense that the present values of y, i.e., $\frac{p}{\rho}$ becomes too large. For $\beta_1 = 1$, we get the present value of y to be $\frac{1}{3}$, but again $r = 1$. In either of the above cases we get accelerating expansion of the Universe. These can be represented diagrammatically in the r, s plane. This is shown in figure 3.1 (taking $A = \frac{1}{3}, \alpha = 1, \beta_1 = 1, \frac{2}{3}, 8\pi G = 1$ and $A = 1, \alpha = 1, \beta_1 = 1, \frac{2}{3}, 8\pi G = 1$). Figure 3.1 represents the evolution of the Universe starting from radiation era to ΛCDM model. Here we get a discontinuity at $\beta_1 = -8\pi G$.

Again for this model

$$\Lambda = \beta_1 \left(\frac{B}{1+A} + \frac{C}{a^{\frac{24\pi G(1+A)(1+\alpha)}{8\pi G + \beta_1}}} \right)^{\frac{1}{1+\alpha}} \tag{3.9}$$

Variation of $\Lambda(t)$ against $a(t)$ is shown in figure 3.2 for different choices of β_1, which represents that regardless the values of β_2, $\Lambda(t)$, i.e., the effect of the cosmological constant decreases with time.

3.3.2 Model with $\Lambda \propto H^2$

Choosing

$$\Lambda(t) = \beta_2 H^2 \tag{3.10}$$

where β_2 is a constant and proceeding as above, we obtain the solutions for ρ, $a(t)$, Λ as,

$$\rho = \left(\frac{B}{1+A} + \frac{C}{a^{(3-\beta_2)(1+A)(1+\alpha)}}\right)^{\frac{1}{1+\alpha}} \tag{3.11}$$

$$a^{f_1 f_2}\ {}_2F_1[f_2, f_2, 1+f_2, -\frac{a^{f_1 B}}{C(1+A)}] = \sqrt{2\pi G}\sqrt{3-\beta_2}\ (1+A)C^{f_2}\ t \tag{3.12}$$

where $f_1 = (3-\beta_2)(1+A)(1+\alpha)$ and $f_2 = \frac{1}{2(1+\alpha)}$

$$\Lambda = \frac{8\pi G\beta_2}{3-\beta_2}\left(\frac{B}{1+A} + \frac{C}{a^{(3-\beta_2)(1+A)(1+\alpha)}}\right)^{\frac{1}{1+\alpha}} \tag{3.13}$$

Here for $\beta_2 < 3$ we can check the consistency of the result by showing $p \simeq A\rho$ at small values of $a(t)$ and $p = -\rho$ for large values of $a(t)$. But if we take $\beta_2 > 3$ we get opposite results which contradict our previous notions of the nature of the EOS (1.61). Again for $\beta_2 = 3$, we get only ΛCDM point ,i.e., we get a discontinuity. Therefore, we restrict our choice for β_2 in this case to be $\beta_2 < 3$.

Computing the state-finder parameters given by equation (1.113), we get the equations for r and s to be,

$$r = 1 + \frac{(3-\beta_2)(1+y)[\{A(1+\alpha) - y\alpha\}(3-\beta_2) - \beta_2]}{2} \tag{3.14}$$

and

$$s = \frac{(3-\beta_2)(1+y)][\{A(1+\alpha) - y\alpha\}(3-\beta_2) - \beta_2]}{3\{(3-\beta_2)y - \beta_2\}} \tag{3.15}$$

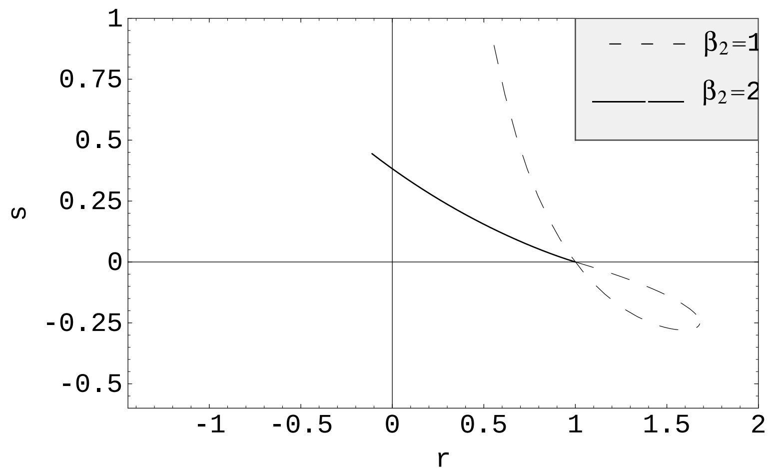

Fig 3.3: The variation of s has been plotted against r for different values of $\beta_2 = 1,\ 2$ and for $\alpha =\ 1$, $A = 1/3$, $8\pi G = 1$.

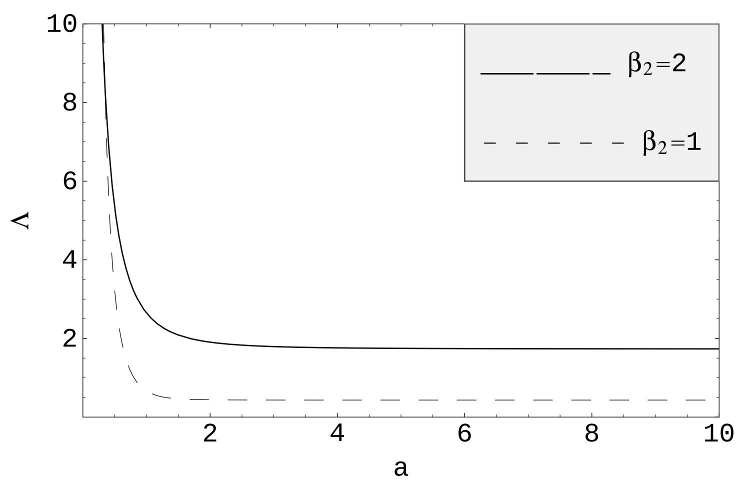

Fig 3.4: The variation of Λ has been plotted against $a(t)$ for different values of $\beta_2 = 1,\ 2$ and for $\alpha =\ 1$, $A = 1/3$, $8\pi G = 1$.

(where $y = \frac{p}{\rho}$), which can still be resolved into a single relation and can be plotted in the r, s plane. Here $q = \frac{1}{2}[(3-\beta_2)y - (\beta_2 - 1)]$. Hence the Universe will accelerate if $q < 0 \Rightarrow y < \frac{\beta_2 - 1}{3-\beta_2}$. Again for the present Universe $q = -\frac{1}{2} \Rightarrow y = \frac{\beta_2 - 2}{3-\beta_2}$. Assuming the present Universe to be dust dominated, i.e., $y = 0$ we get the best fit value for β_2 to be 2. Taking $A = \frac{1}{3}, \alpha = 1, 8\pi G = 1, \beta_2 = 2$ and $y = 0$ (i.e., dust dominated present Universe) we get the present value to be $r = 1/3$, also the same values with $\beta_2 = 1$ gives the present values to be $y = -\frac{1}{2}$, $r = \frac{5}{3}$ .This is shown in figure 3.3 ($A = \frac{1}{3}, \alpha = 1, \beta_2 = 1$ and $2, 8\pi G = 1$), which explains the evolution of the Universe from radiation era to ΛCDM model. Again variation of Λ against time is shown in figure 3.4, where we can see that Λ decreases with time for whatever the value of β_2 be.

3.3.3 Model with $\Lambda \propto \frac{\ddot{a}}{a}$

Taking

$$\Lambda = \beta_3 \frac{\ddot{a}}{a} \tag{3.16}$$

(where β_3 is a constant), and proceeding as above we get a relation for ρ as,

$$\rho^{(\frac{2}{1+A} - \beta_3)} \left(1 + A - \frac{B}{\rho^{\alpha+1}}\right)^{(\frac{2}{(1+A)(1+\alpha)} - \beta_3)} = \frac{C}{a^{2(3-\beta_3)}} \tag{3.17}$$

Unlike the previous two cases here we get a far more restricted solution. Here the only choice of β_3 for which we get the feasible solution satisfying $p \simeq A\rho$ for small values of $a(t)$ and $p \simeq -\rho$ for large values of $a(t)$ is

$$\beta_3 < \frac{2}{(1+A)(1+\alpha)} \qquad \text{or} \qquad \beta_3 > 3 \tag{3.18}$$

Again since $q = -\frac{\ddot{a}}{aH^2} = -\frac{\Lambda}{\beta_3 H^2} = \frac{4\pi G(\rho+3p)}{(3-\beta_3)H^2} = \frac{4\pi G(\rho+3p)}{(3-\beta_3)H^2}$, $\beta_3 > 3$ implies $q < 0$ without even violating the energy-condition $\rho + 3p \geq 0$. Although $\beta_3 < \frac{2}{(1+A)(1+\alpha)}$ causes the acceleration of the Universe violating the energy-condition. Taking $q = -\frac{1}{2}$ for the

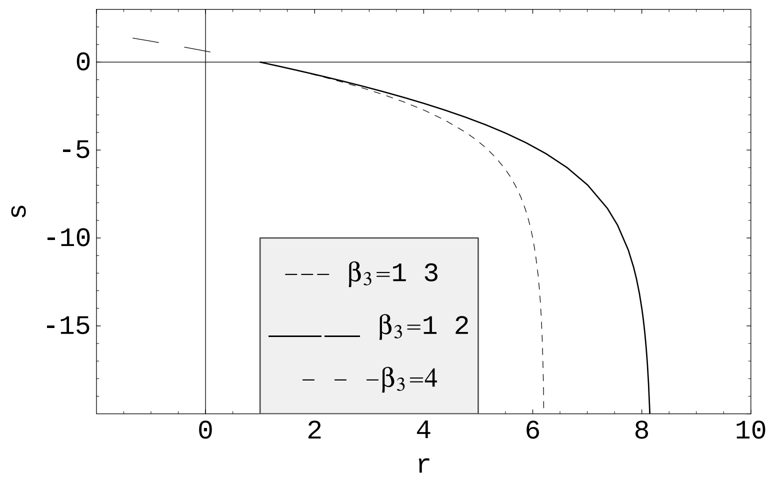

Fig 3.5: The variation of s has been plotted against r for different values of $A=\frac{1}{3},\alpha=1, 8\pi G=1, \beta_3=\frac{1}{2}, 4$ and $\frac{1}{3}$

present epoch, we obtain $y=\frac{(\beta_3-4)}{(6-\beta_3)}$. Hence the present epoch is dust filled if $\beta_3=4$ and thus giving the present value of r to be $-\frac{1}{7}$ for $A=\frac{1}{3},\ \alpha=1,\ 8\pi G=1$. On using relation (3.18), ρ and therefore $a,\ \Lambda$ cannot be expressed in an open form. We can rather derive a solution for Λ in terms of $p,\ \rho$ as,

$$\Lambda=\frac{4\pi G\beta_3}{\beta_3-3}(\rho+3p) \tag{3.19}$$

Using equations (1.113) we get the statefinder parameters as,

$$r=1-\frac{(1+y)(\beta_3-3)[\beta_3+(\beta_3+6)x]}{(\beta_3+\beta_3 x-2)(\beta_3+\beta_3 y-2)} \tag{3.20}$$

and

$$s=\frac{2(1+y)(\beta_3-3)[\beta_3+(\beta_3+6)x]}{[\beta_3+(\beta_3+6)y][\beta_3+\beta_3 x-2]} \tag{3.21}$$

where $y=\frac{p}{\rho}$ and $x=\frac{\partial p}{\partial \rho}$, i.e., $x=A(1+\alpha)-y\alpha$ [from equation (1.61)].

Eliminating y between the equations (3.20) and (3.21), we get a single relation of r and s, which can be represented diagrammatically in the r,s plane (figure 3.5). Here

we have taken $A = \frac{1}{3}, \alpha = 1, 8\pi G = 1, \beta_3 = \frac{1}{2}, 4$ and $\frac{1}{3}$, combining two cases. Taking $\beta_3 = \frac{1}{2}, \frac{1}{3}$ we can explain the evolution of the Universe starting from $\frac{p}{\rho} = -\frac{1}{3}$ to ΛCDM model and $\beta_3 = 4$ explains the evolution of the Universe starting from radiation era to $y = -\frac{1}{3}$, as seen from the expression for q. Considering the present epoch to be dust-dominated, the present value of r is given for $\beta_3 = 4$ to be $-\frac{1}{7}$. As follows, the former two cases cannot give the present value of r, as $y = 0 > -\frac{1}{3}$ for the present epoch. Here we have an infinite discontinuity at $\frac{p}{\rho} = -\frac{1}{3}$, i.e., when $\rho+3p = 0$. Also since we do not get a closed from of ρ here, it is difficult to plot Λ against the scale factor $a(t)$.

3.4 Models with G and Λ both variable

Now we consider G as well as Λ to be variable. With this the conservation law reads,

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{3.22}$$

and

$$\dot{\Lambda} + 8\pi \dot{G}\rho = 0 \tag{3.23}$$

Now we study the various phases of the Universe represented by these models.

Equation (3.22) together with equation (1.60) yield the solution for ρ as,

$$\rho = \left(\frac{B}{1+A} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right)^{\frac{1}{1+\alpha}} \tag{3.24}$$

where C is an arbitrary constant. This result is consistent with the results already obtained [Gorini etal, 2003].

3.4.1 Model with $\Lambda \propto \rho$

Here we consider

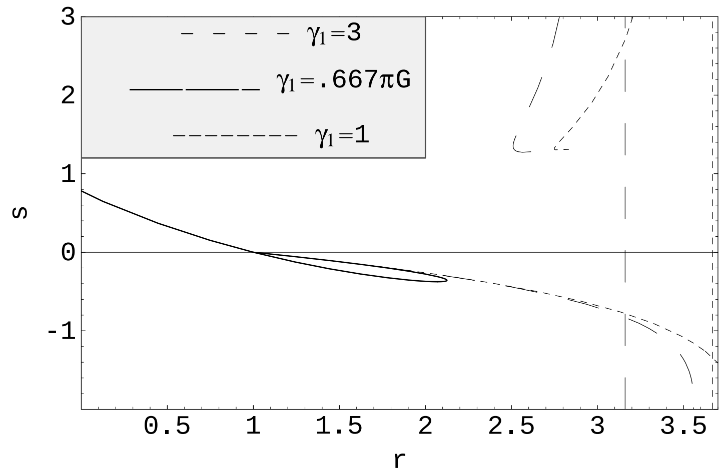

Fig 3.6: The variation of s has been plotted against r for different values of $\gamma_1 = 1, 3$ and 3.5 and $A = \frac{1}{3}, \alpha = 1, B = 1, C_1 = 1$

$$\Lambda = \gamma_1\ \rho \tag{3.25}$$

where γ_1 is a constant.

Equation (3.22), (3.23) and (3.25) give,

$$G = C_1 - \frac{\gamma_1}{8\pi} \log \rho \tag{3.26}$$

where C_1 is a constant and ρ is given by equation (3.24).

Using equations (1.113), (3.1), (3.23) and (3.26), we get,

$$\begin{aligned} G &= C_1 + \tfrac{\gamma_1(1+\alpha)}{8\pi} \log(\tfrac{B}{A-y}) \\ r &= 1 + \tfrac{9(1+y)[8\pi G\{A(1+\alpha)-y\alpha\}-\gamma_1(1+y)]}{2(8\pi G+\gamma_1)} \\ s &= \tfrac{(1+y)[8\pi G\{A(1+\alpha)-y\alpha\}-\gamma_1(1+y)]}{(8\pi G y-\gamma_1)} \end{aligned} \tag{3.27}$$

where $y = \frac{p}{\rho}$.

Equation (3.27) cannot be resolved to get a single relation between r and s, rather we obtain a parametric relation between the same with $y = \frac{p}{\rho}$ as the parameter. This can be represented diagrammatically in the r, s plane, which is shown in figure 3.6 taking $\gamma_1 = 1, 3$ and 3.5 and $A = \frac{1}{3}, \alpha = 1, B = 1, C_1 = 1$. Now $q = \frac{4\pi G(1+3y)-\gamma_1}{8\pi G+\gamma_1}$. Taking into account that $q = -\frac{1}{2}$ for the present epoch, we get $y = \frac{\gamma_1}{8\pi G} - \frac{2}{3}$. Therefore, for the present dust-dominated era $y = 0$ and $\gamma_1 = \frac{16\pi G}{3}$. hence for the This models represents the Universe starting from the radiation era to ΛCDM model. Again figure 3.7 represents the variation of Λ against the scale factor $a(t)$ with $\gamma_1 = 1, 3, 3.5$ and figure 3.8 represents the variation G against the scale factor $a(t)$. These figures show that for this particular phenomenological model of Λ, G starting from very low initial value increases largely and becomes constant after a certain period of time, whereas Λ starting from a very large decreases largely to reach a very low value and becomes constant.

3.4.2 Model with $\Lambda \propto H^2$

We consider

$$\Lambda = \gamma_2 H^2 \tag{3.28}$$

Proceeding as above we get,

$$\Lambda = 8\pi G \frac{\gamma_2}{3-\gamma_2}\rho \tag{3.29}$$

where γ_2 is a constant.

Solving equation (3.22), (3.23) and (3.29) we get,

$$G = \frac{C_2}{\rho^{\frac{\gamma_2}{3}}} \tag{3.30}$$

where C_2 is a constant.

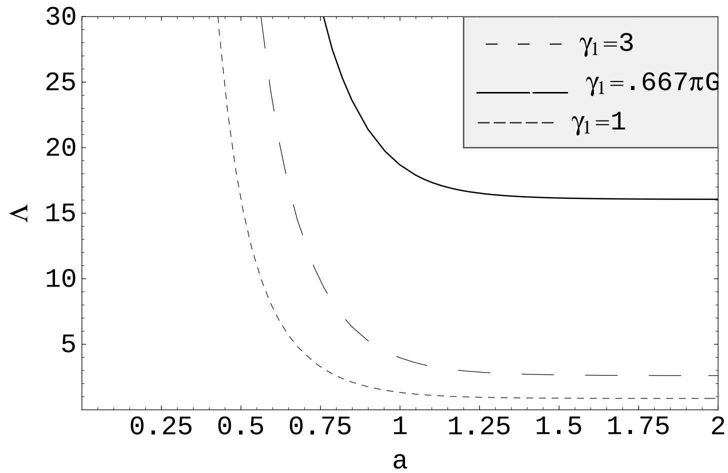

Fig 3.7: The variation of Λ is plotted against $a(t)$ for different values of $\gamma_1 = 1, 3, 3.5$ and for $\alpha = 1$, $A = 1/3$, $C_1 = 1$.

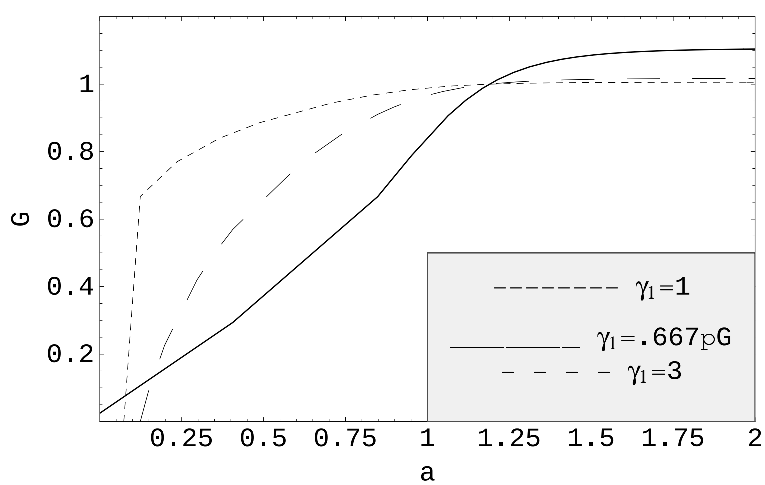

Fig 3.8: The variation of G is plotted against $a(t)$ for different values of $\gamma_1 = 1, 3, 3.5$ and for $\alpha = 1$, $A = 1/3$, $C_1 = 1, B = 1$, $C = 1$.

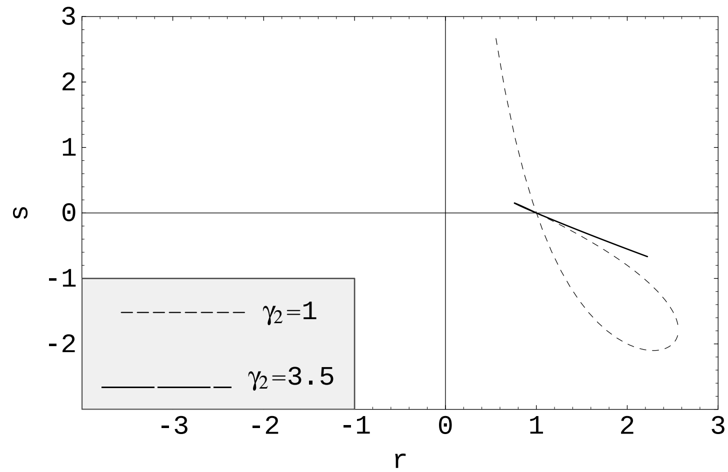

Fig 3.9: The variation of s is plotted against r for different values of $\gamma_2 = 1$ and 3.5 and $A = \frac{1}{3}, \alpha = 1, B = 1, C_2 = 1,\ C = 1$.

Using equations (1.113), (3.1) and (3.23), we find the state-finder parameters as,

$$r = 1 + \frac{(1+y)(3-\gamma_2)[3\{A(1+\alpha) - y\alpha\} - (1+y)\gamma_2]}{2} \tag{3.31}$$

and

$$s = \frac{(1+y)(3-\gamma_2)[3\{A(1+\alpha) - y\alpha\} - (1+y)\gamma_2]}{(3-\gamma_2)y - \gamma_2} \tag{3.32}$$

where $y = \frac{p}{\rho}$.

Now $q = \frac{1}{2}[(3-\gamma_2)y - (\gamma_2 - 1)]$. These equations can further be resolved into a single relation of r and s, which can be plotted diagrammatically in the r, s plane. Here we get a discontinuity at $\gamma_2 = 3$. We have plotted these values in the r, s plane taking $\gamma_2 = 1$ and 3.5 in figure 3.9 ($A = \frac{1}{3}, \alpha = 1$). This case explains the present acceleration of the Universe, starting from radiation era to ΛCDM model.

Also figures 3.10 and 3.11 show respectively the variation of G and Λ against the scale factor for the same values of the constants. Here also like the previous case G starting from a very low initial value increases largely and then continues to be constant near

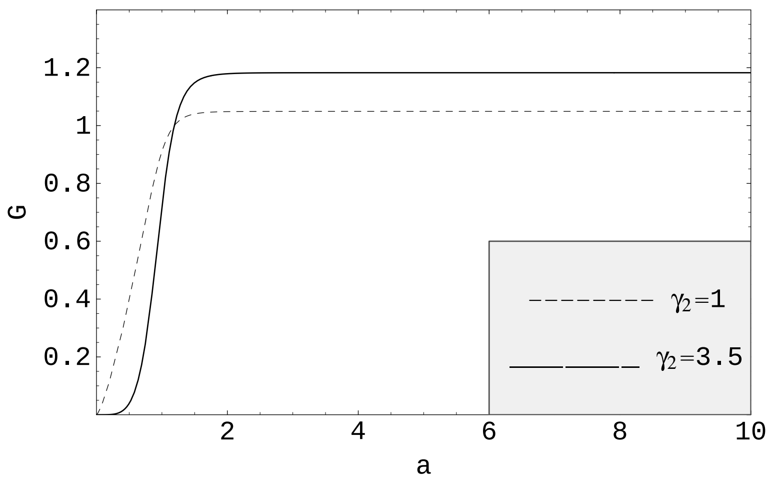

Fig 3.10: The variation of G is plotted against $a(t)$ for different values of $\gamma_1 = 1, 3.5$ and for $\alpha = 1$, $A = 1/3$, $C_1 = 1$, $C = 1$, $B = 1$.

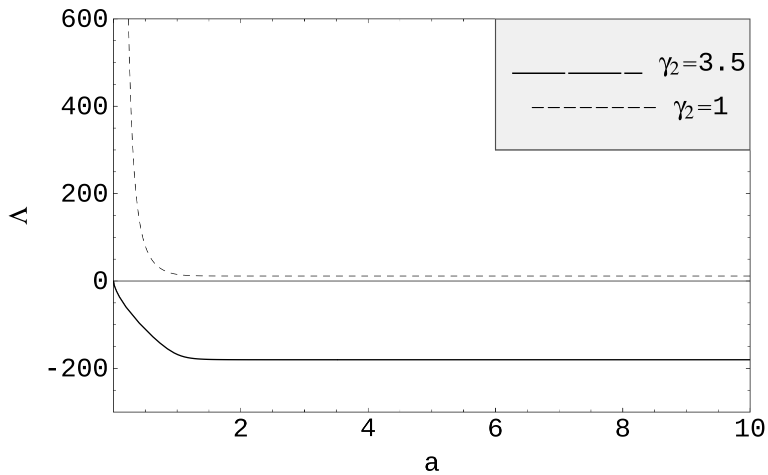

Fig 3.11: The variation of Λ is plotted against $a(t)$ for different values of $\gamma_1 = 1, 3, 3.5$ and for $\alpha = 1$, $A = 1/3$, $C_2 = 1, B = 1$, $C = 1$.

unity. On the other hand Λ starting from a large value decreases largely and continues to be constant after a certain period of time.

3.4.3 Model with $\Lambda \propto \frac{\ddot{a}}{a}$

Here we consider

$$\Lambda = \gamma_3 \frac{\ddot{a}}{a} \tag{3.33}$$

where γ_3 is a constant.

Using equation (3.33) in equations (3.1) and (3.2), we get,

$$\Lambda = -\frac{4\pi G \gamma_3}{3-\gamma_3}(\rho + 3p) \tag{3.34}$$

Also, G can be solved to be,

$$G = C_3[\rho^{\frac{1+3A}{2-\gamma_3(1+A)}}\{2-\gamma_3(1+A)+\frac{B\gamma_3}{\rho^{\alpha+1}}\}^{\{-\frac{3\alpha}{\gamma_3(1+\alpha)}+\frac{1+3A}{(1+\alpha)(2-\gamma_3(1+A))}\}}]^{\frac{\gamma_3}{3}} \tag{3.35}$$

Using equations (1.113), (3.1), (3.22), (3.23), we find,

$$r = 1 + \frac{(1+y)(3-\gamma_3)[6\{A(1+\alpha)-y\alpha\}+\gamma_3(1+y)]}{[2-\gamma_3(1+y)]^2} \tag{3.36}$$

and

$$s = \frac{2(1+y)(3-\gamma_3)[6\{A(1+\alpha)-y\alpha\}+\gamma_3(1+y)]}{3[2-\gamma_3(1+y)][\gamma_3+(\gamma_3+6)y} \tag{3.37}$$

where $y = \frac{p}{\rho}$ and C_3 is a constant. Equations (3.36) and (3.37) can further be resolved to get one single relation between r and s and plotted diagrammatically taking $\gamma_3 = 2$ and 3.5 (figure 3.12). Since deceleration parameter $q = -\frac{\ddot{a}}{aH^2} = -\frac{\lambda}{\gamma_3 H^2} = \frac{4\pi G}{(3-\gamma_3)H^2}$, is negative in the present epoch, we get $3-\gamma_3 < 0$, i.e., $\gamma_3 > 3$. Also for $\gamma_3 = 3$ we

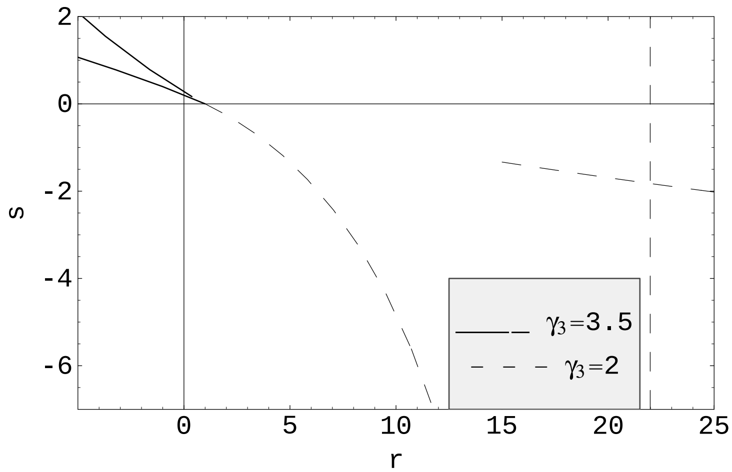

Fig 3.12: The variation of s is plotted against r for different values of $\gamma_2 = 2$ and 3.5 and $A = \frac{1}{3}, \alpha = 1, B = 1, C_3 = 1,\ C\ =\ 1$.

get discontinuity. Both the models represent the phases of the Universe starting from radiation era to ΛCDM model. Again G and Λ can be plotted against a (figures 3.13 and 3.14 respectively) . Unlike the previous cases this model an opposite nature of G and Λ, as G decreases with time and Λ increases with time.

3.5 Discussion

Here we have considered three phenomenological models of Λ, with or without keeping G to be constant. Keeping G constant we always get accelerated expansion of the Universe. For the first case, i.e., $\Lambda \propto \rho$ or more precisely, $\Lambda = \beta_1 \rho$, for particular choices of the constants we get that the dark energy responsible for the present acceleration is nothing but Λ. Also the density parameter of the Universe for this case is given by, $\Omega_m^{\beta_1} = \frac{8\pi G\rho}{3H^2} = \frac{8\pi G}{8\pi G+\beta_1}$ and the vacuum density parameter is $\Omega_\Lambda^{\beta_1} = \frac{\Lambda}{3H^2} = \frac{\beta_1}{8\pi G+\beta_1}$, so that $\Omega_{total} = \Omega_m + \Omega_\Lambda = \Omega_m^{\beta_1} + \Omega_\Lambda^{\beta_1} = 1$. Also for $\Lambda \propto H^2$, i.e., $\Lambda = \beta_2\ H^2$, the density parameter and vacuum density parameter are given by, $\Omega_m^{\beta_2} = \frac{3-\beta_2}{3}$ and $\Omega_\Lambda^{\beta_1} = \frac{\beta_2}{3}$ respectively, so that $\Omega_{total} = \Omega_m^{\beta_2} + \Omega_\Lambda^{\beta_1} = 1$. Again for $\Lambda \propto \frac{\ddot{a}}{a}$ or $\Lambda = \beta_3\ \frac{\ddot{a}}{a}$, we have the corresponding parameters as, $\Omega_m^{\beta_3} = \frac{2(3-\beta_3)}{3(2-\beta_3-\beta_3\frac{p}{\rho})}$, $\Omega_\Lambda^{\beta_3} = \frac{-\beta_3(1+3\frac{p}{\rho})}{3(2-\beta_3-\beta_3\frac{p}{\rho})}$ and $\Omega_{total} = 1$. Now $\Omega_{total} = \Omega_m^{\beta_3} + \Omega_\Lambda^{\beta_3} = 1$ for all the models. Also we can compare these models by

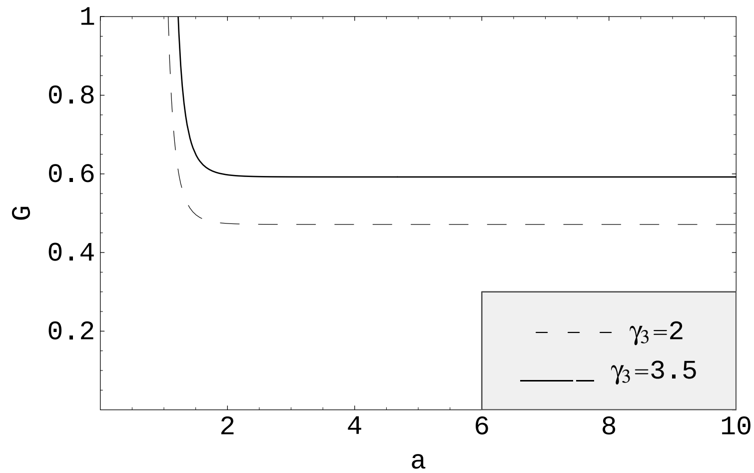

Fig 3.13: The variation of G is plotted against $a(t)$ for different values of $\gamma_1 = 2, 3.5$ and for $\alpha = 1$, $A = 1/3$, $C_3 = 1$, $C = 1$, $B = 1$.

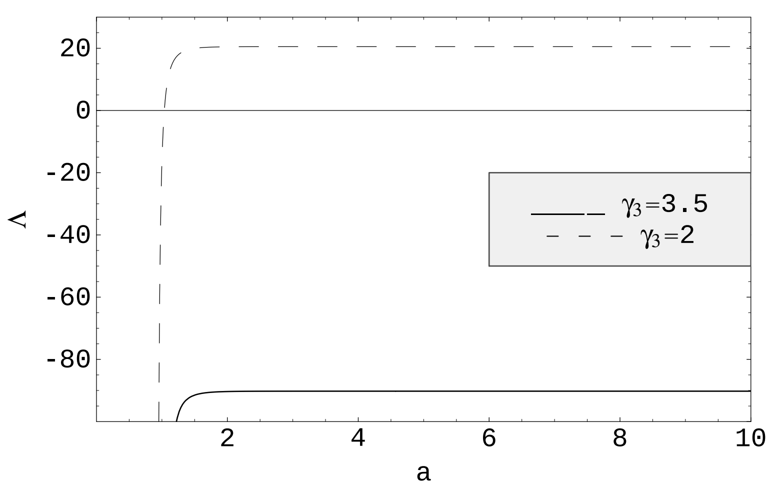

Fig 3.14: The variation of Λ is plotted against $a(t)$ for different values of $\gamma_1 = 2, 3, 3.5$ and for $\alpha = 1$, $A = 1/3$, $C_3 = 1, B = 1$, $C = 1$.

taking, $\Omega_m^{\beta_1} = \Omega_m^{\beta_2}$, so that $\beta_2 = \frac{3\beta_1}{(8\pi G+\beta_1)}$. Now we would like to take into account the present values of the density parameter and vacuum parameter obtained by the recent measurements. Considering $\Omega_{m0} = 0.33 \pm .035$, we calculate the present values of the proportional constants to be $1.7397K \leq {\beta_1}^0 \leq 2.3898K$, $1.905 \leq {\beta_2}^0 \leq 2.115$ and $3.7937 \leq {\beta_3}^0 \leq 4.2099$, where $K = 8\pi G_0$ and G_0 is the present value of the gravitational constant. Thus we get the value of ${\beta_3}^0$ to be lesser than the previous works. Again considering G to be time-dependent, we get the same values of the parameters as that with G constant, i.e., the ranges of ${\gamma_1}^0, {\gamma_2}^0, {\gamma_3}^0$ are same as that of ${\beta_1}^0, {\beta_2}^0, {\beta_3}^0$ respectively. Here also we get cosmic acceleration and the nature of variation G and Λ as well. We get two different cases regarding the variation of G and Λ. For the first two cases we see that G increases and Λ decreases with time, whereas for the third case G decreases and Λ increases with time. In all the cases the values become constant after a certain period of time, i.e.,the present day values of G and Λ are constants. Thus these models with the phenomenological laws give us some interesting features of the cosmic acceleration and some modified values of the parameters. Also we get the natures of the Cosmological Constant and the Gravitational Constant over the total age of the Universe. We can also make use of the statefinder parameters to show the evolution of the Universe starting from radiation era to ΛCDM model.

Chapter 4

Generalized Cosmic Chaplygin Gas Model

4.1 Prelude

Recently developed Generalized Cosmic Chaplygin gas (GCCG) is studied as an unified model of dark matter and dark energy. To explain the recent accelerating phase, the Universe is assumed to have a mixture of radiation and GCCG. The mixture is considered for without or with interaction. Solutions are obtained for various choices of the parameters and trajectories in the plane of the statefinder parameters and presented graphically.

In 2003, Gonzálex-Diaz have introduced the generalized cosmic Chaplygin gas (GCCG) model in such a way that the resulting models can be made stable and free from unphysical behaviours even when the vacuum fluid satisfies the phantom energy condition. The EOS of this model is

$$p = -\rho^{-\alpha}\left[C + (\rho^{1+\alpha} - C)^{-\omega}\right] \tag{4.1}$$

where $C = \frac{A}{1+\omega} - 1$ with A a constant which can take on both positive and negative values and $-l < \omega < 0$, l being a positive definite constant which can take on values larger than unity.

The EOS reduces to that of current Chaplygin unified models for dark matter and

dark energy in the limit $\omega \to 0$ and satisfies the conditions: (i) it becomes a de Sitter fluid at late time and when $\omega = -1$, (ii) it reduces to $p = w\rho$ in the limit that the Chaplygin parameter $A \to 0$, (iii) it also reduces to the EOS of current Chaplygin unified dark matter models at high energy density and (iv) the evolution of density perturbations derived from the chosen EOS becomes free from the pathological behaviour of the matter power spectrum for physically reasonable values of the involved parameters at late time. This EOS shows dust era in the past and ΛCDM in the future.

In this chapter, we consider the Universe is filled with the mixture of radiation and GCCG. We also perform a statefinder diagnostic to this model without and with interaction in different cases.

4.2 GCCG in presence of radiation

The metric of a spatially flat isotropic and homogeneous Universe in FRW model is equation (1.7). The Einstein field equations are (choosing $8\pi G = c = 1$)

$$3\frac{\dot{a}^2}{a^2} = \rho_{tot} \tag{4.2}$$

and

$$6\frac{\ddot{a}}{a} = -(\rho_{tot} + 3p_{tot}) \tag{4.3}$$

The energy conservation equation ($T^{\nu}_{\mu;\nu} = 0$) is

$$\dot{\rho}_{tot} + 3\frac{\dot{a}}{a}(\rho_{tot} + p_{tot}) = 0 \tag{4.4}$$

where, ρ_{tot} and p_{tot} are the total energy density and the pressure of the Universe, given by,

$$\rho_{tot} = \rho + \rho_r \tag{4.5}$$

and

$$p_{tot} = p + p_r \tag{4.6}$$

with ρ and p are respectively the energy density and pressure due to the GCCG satisfying the EOS (4.1) and ρ_r and p_r are the energy density and the pressure corresponding to the radiation fluid with EOS,

$$p_r = \gamma \rho_r \tag{4.7}$$

where $\gamma = \frac{1}{3}$.

Since GCCG can explain the evolution of the Universe starting from dust era to ΛCDM, considering the mixture of GCCG with radiation would make it possible to explain the evolution of the Universe from radiation to ΛCDM.

4.2.1 Non-interacting model

In this case GCCG and the radiation fluid are conserved separately. Conservation equation (4.4) yields,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \tag{4.8}$$

and

$$\dot{\rho_r} + 3\frac{\dot{a}}{a}(\rho_r + p_r) = 0 \tag{4.9}$$

From equations (4.1), (4.7), (4.8), (4.9) we have

$$\rho = \left[C + \left(1 + \frac{B}{a^{3(1+\alpha)(1+\omega)}}\right)^{\frac{1}{1+\omega}}\right]^{\frac{1}{1+\alpha}} \tag{4.10}$$

and

$$\rho_r = \rho_0 \; a^{-3(1+\gamma)} \tag{4.11}$$

For the two component fluids, statefinder parameters (1.113) takes the following forms:

$$r = 1 + \frac{9}{2(\rho+\rho_r)}\left[\frac{\partial p}{\partial \rho}(\rho+p) + \frac{\partial p_r}{\partial \rho_r}(\rho_r+p_r)\right] \tag{4.12}$$

and

$$s = \frac{1}{(p+p_r)}\left[\frac{\partial p}{\partial \rho}(\rho+p) + \frac{\partial p_r}{\partial \rho_r}(\rho_r+p_r)\right] \tag{4.13}$$

Also the deceleration parameter q has the form:

$$q = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} + \frac{3}{2}\left(\frac{p+p_r}{\rho+\rho_r}\right) \tag{4.14}$$

Now substituting $u = \rho^{1+\alpha}, \; y = \frac{\rho_r}{\rho}$, equation (4.12) and (4.13) can be written as,

$$r = 1 + \frac{9}{2(1+y)}\left[\left(1 - \frac{C}{u} - \frac{(u-C)^{-\omega}}{u}\right)\{\frac{\alpha C}{u} + \frac{\alpha}{u}(u-C)^{-\omega}\right.$$
$$\left. + \omega(1+\alpha)(u-C)^{-\omega-1}\} + \gamma(1+\gamma)y\right] \tag{4.15}$$

and

$$s = \frac{2(r-1)(1+y)}{9\left[\gamma y - \frac{C}{u} - \frac{(u-C)^{-\omega}}{u}\right]} \tag{4.16}$$

Normalizing the parameters we have shown the graphical representation of the $\{r, s\}$ parameters in figure 4.1.

4.2.2 Interacting Model

We consider the GCCG interacting with radiation fluid through an energy exchange between them. The equations of motion can be written as,

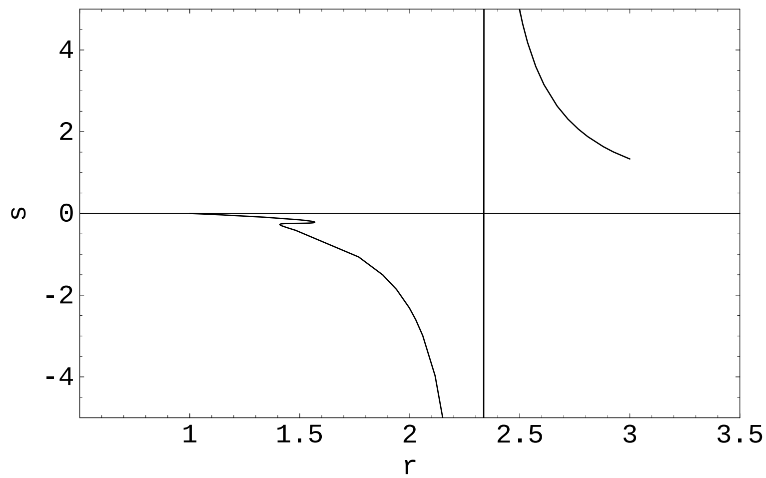

Fig 4.1: The variation of s is plotted against r for $C = 1, B = 1, \alpha = 1, \omega = -2, \rho_0 = 1$.

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -3H\delta \tag{4.17}$$

and

$$\dot{\rho_r} + 3\frac{\dot{a}}{a}(\rho_r + p_r) = 3H\delta \tag{4.18}$$

where δ is a coupling function.

Let us choose,

$$\delta = \epsilon\frac{(\rho^{1+\alpha} - C)^{-\omega}}{\rho^{\alpha}} \tag{4.19}$$

Now equation (4.17) together with equation (4.1) gives,

$$\rho = \left[C + \left(1 - \epsilon + Ba^{3-(1+\alpha)(1+\omega)}\right)^{\frac{1}{1+\omega}}\right]^{\frac{1}{1+\alpha}} \tag{4.20}$$

Also equations (4.7), (4.18) and (4.20) give

$$\rho_r = \rho_0\ a^{-3(1+\gamma)} + 3\ \epsilon\ a^{-3(1+\gamma)} I \tag{4.21}$$

with

$$I=-\frac{1}{3B(1+\alpha)}\int\frac{dx}{(C+x)^{\frac{\alpha}{(1+\alpha)}}}\left\{\frac{x^{1+\omega}+\epsilon-1}{B}\right\}^{-\frac{1+\gamma}{(1+\omega)(1+\alpha)}-1} \tag{4.22}$$

and

$$x=\left[1-\epsilon+Ba^{-3(1+\omega)(1+\alpha)}\right]^{\frac{1}{1+\omega}} \tag{4.23}$$

From (4.20), we see that if $\epsilon=0$, i.e., $\delta=0$, then the expression (4.20) reduces to the expression (4.10).

Now for the two component interacting fluids with equations of motion (4.17) and (4.18), the r,s parameters read:

$$r=1+\frac{9}{2(\rho+\rho_r)}\left[\frac{\partial p}{\partial\rho}(\rho+p+\delta)+\frac{\partial p_r}{\partial\rho_r}(\rho_r+p_r-\delta)\right] \tag{4.24}$$

and

$$s=\frac{2(r-1)(\rho+\rho_r)}{9(p+p_r)} \tag{4.25}$$

Also the deceleration parameter q has the form:

$$q=-\frac{1}{2}\left(1+3\frac{p+p_r}{\rho+\rho_r}\right) \tag{4.26}$$

Now substituting $u=\rho^{1+\alpha},\ y=\frac{\rho_r}{\rho}$, equation (4.12) and (4.13) can be written as,

$$r=1+\frac{9}{2(1+y)}\left[\frac{\partial p}{\partial\rho}\left(1+\frac{p}{\rho}+\frac{\delta}{\rho}\right)+\gamma\left\{(1+\gamma)y-\frac{\delta}{\rho}\right\}\right] \tag{4.27}$$

and

$$s=\frac{2(r-1)(1+y)}{9\left(\frac{p}{\rho}+\gamma y\right)} \tag{4.28}$$

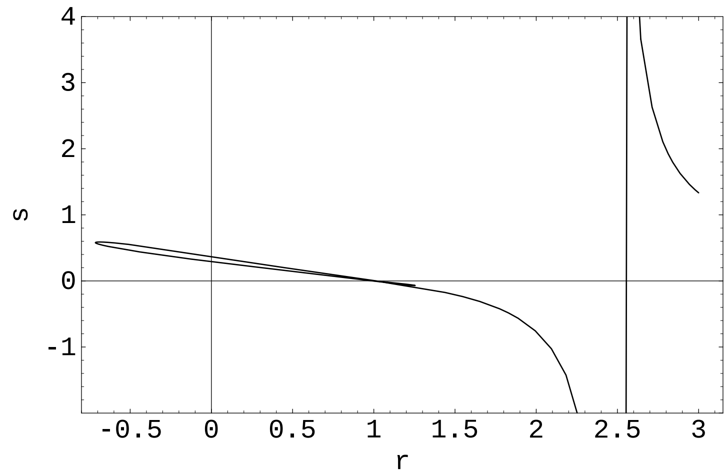

Fig 4.2: The variation of s is plotted against r for $C = C_0 = B = 1, \alpha = 1, \omega = -2, \rho_0 = 1, \epsilon = \frac{1}{2}$.

where,

$$u = \left[C + \left(1 - \epsilon + Ba^{3-(1+\alpha)(1+\omega)}\right)^{\frac{1}{1+\omega}}\right]$$

$$y = \frac{\rho_0}{\rho}\ a^{-3(1+\gamma)} + 3\ \frac{\epsilon}{\rho}\ a^{-3(1+\gamma)} I$$

$$\frac{p}{\rho} = -\frac{1}{u}\{C + (u - C)^{-\omega}\}$$

$$\frac{\delta}{\rho} = \epsilon \frac{(u - C)^{-\omega}}{u}$$

and

$$\frac{\partial p}{\partial \rho} = \frac{\alpha C}{u} + \frac{\alpha}{u}(u - C)^{-\omega} + \omega(1 + \alpha)(u - C)^{-\omega - 1}$$

Now we find the exact solution for the r, s parameters for the following particular choices of ω:

(i) If $-\frac{(1+\gamma)}{(1+\omega)(1+\alpha)} - 1 = 0$, i.e., $\omega = \frac{-2-\gamma-\alpha}{1+\alpha}$, equation (4.21) can be written as

$$\rho_r = \rho_0\ a^{-3(1+\gamma)} - \frac{\epsilon}{B} a^{-3(1+\gamma)} \rho \tag{4.29}$$

as $I = -\frac{1}{3B}(c + x)^{\frac{1}{1+\alpha}}$

Normalizing the parameters, the corresponding statefinder parameters are given in figure 4.2.

(ii) If $-\frac{(1+\gamma)}{(1+\omega)(1+\alpha)} - 1 = 1$, i.e., $\omega = \frac{-3-\gamma-2\alpha}{2(1+\alpha)}$, equation (4.21) can be written as

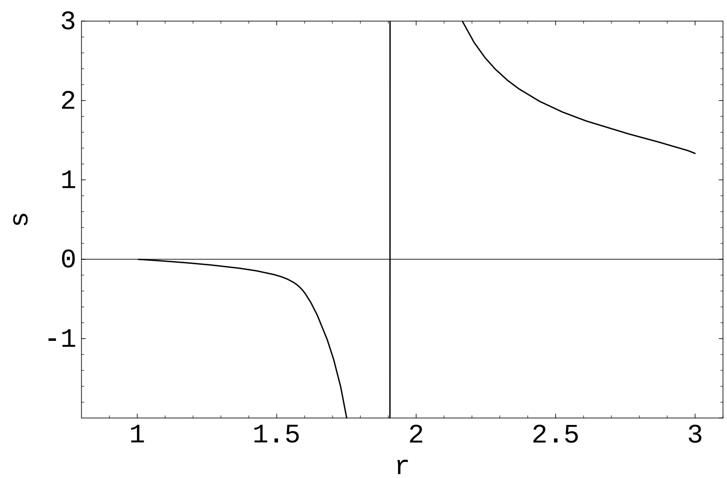

Fig 4.3: The variation of s is plotted against r for $C = C_0 = B = 1, \alpha = 1, \omega = -\frac{3}{2}, \rho_0 = 1, \epsilon = \frac{1}{2}$.

$$\rho_r = \rho_0\, a^{-3(1+\gamma)} - \frac{\epsilon(\epsilon-1)}{B^2} a^{-3(1+\gamma)} - \frac{\epsilon a^{-3(1+\gamma)}}{B^2(1+\alpha)(2+\omega)C^{\frac{\alpha}{1+\alpha}}} x^{2+\omega}\, {}_2F_1[2+\omega, \frac{\alpha}{1+\alpha}, 3+\omega, -\frac{x}{C}] \tag{4.30}$$

Normalizing the parameters, the corresponding statefinder parameters are given in figure 4.3.

(iii) If $\omega = -2$, equation (4.21) can be written as

$$\rho_r = \rho_0\, a^{-3(1+\gamma)} - \frac{\epsilon}{(1+2\alpha-\gamma)} \frac{a^{-3(1+\gamma)}}{x^{\frac{1+2\alpha-\gamma}{(1+\alpha)}}} \frac{B^{-\frac{1+\gamma}{(1+\alpha)}}}{C^{\frac{\alpha}{1+\alpha}}}$$

$$AppellF_1\left[\frac{1+2\alpha-\gamma}{(1+\alpha)}, \frac{\alpha}{1+\alpha}, \frac{\alpha-\gamma}{(1+\alpha)}, \frac{2+3\alpha-\gamma}{(1+\alpha)}, -\frac{x}{C}, x - x\epsilon\right] \tag{4.31}$$

Normalizing the parameters, the corresponding statefinder parameters are given in figure 4.4.

4.3 Discussion

Recently developed Generalized Cosmic Chaplygin gas (GCCG) is studied as an unified model of dark matter and dark energy. In this chapter, we have considered the matter in our Universe as a mixture of the GCCG and radiation as GCCG can explain the evolution

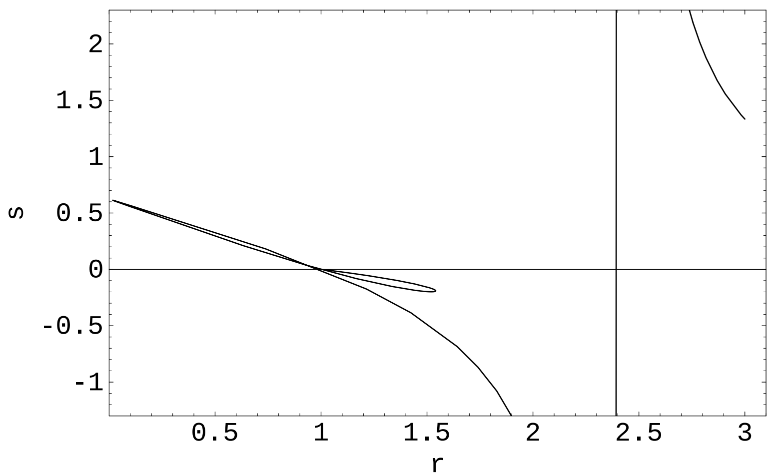

Fig 4.4: The variation of s is plotted against r for $C = C_0 = B = 1, \alpha = 1, \rho_0 = 1, \epsilon = \frac{1}{2}$.

of the Universe from dust era to ΛCDM. These gases are taken both as non-interacting and interacting mixture. In the first case we have considered a non-interacting model and plotted the r, s parameters. As expected this model represents the evolution of the Universe from radiation era to ΛCDM with a discontinuity at $r = 2$ where it represents the dust era (for $r = 2$ implies the dust era, $r = 1$ implies ΛCDM, $r < 1$ phantom). In the second case the interaction term is chosen in a very typical form to solve the corresponding conservation equations analytically. Also the statefinder parameters are evaluated for various choices of parameters and the trajectories in the $\{r, s\}$ plane are plotted to characterize different phases of the Universe. These trajectories show discontinuity at same r in the neighbourhood of $r = 2$ and have peculiar behaviour around $r = 1$. The $\{r, s\}$ curves have two branches on two sides of the asymptote. The branch on the right hand side of the asymptote corresponds to decelerating phase before (or up to) dust era, while the left hand side branch has a transition from decelerating phase upto ΛCDM era. Some peculiarity has been shown in figures 2 and 4 around $r = 1$. In these two cases, the model goes further from ΛCDM to phantom era and then back to ΛCDM. Moreover, in figure 4.4, there is further transition from ΛCDM to decelerating phase and then then again back to ΛCDM. Thus we can conclude that the present model describes a number of transitions from decelerating to accelerating phase and vice-versa.

Chapter 5

Variable Modified Chaplygin Gas Model in presence of Scalar Field

5.1 Prelude

We have already studied the properties of GCG and MCG and their roles in explaining the evolution of the Universe. Later Sthi etal [2006] and Guo etal [2007] introduced inhomogeneity in the EOS of MCG given by equation (1.60) by considering B to be a function of the scale factor $a(t)$. This assumption is reasonable since $B(a)$ is related to the scalar potential if we take the Chaplygin gas as a Born-Infeld scalar field [Bento etal, 2003].

In this chapter we generalize the above model and present a new form of the well known Chaplygin gas model by introducing inhomogeneity in the EOS by considering both A and B in the EOS (1.60) to be a function of the scale factor $a(t)$. We show that this model can explain the evolution of the Universe suitably by choosing different values of the parameters and also can explain $\omega = -1$ crossing.

We have also seen that interaction models where the dark energy weakly interacts with the dark matter have been studied to explain the evolution of the Universe. These models describe an energy flow between the components. To obtain a suitable evolution of the Universe the decay rate should be proportional to the present value of the Hubble

parameter for good fit to the expansion history of the Universe as determined by the Supernovae and CMB data [Berger and Shojaei, 2006]. A variety of interacting dark energy models have been proposed and studied for this purpose [Zimdahl, 2005; Cai and Wang, 2005]. We therefore have also considered a interaction of this model with the scalar field by introducing a phenomenological coupling function which describes the energy flow between them, thus showing the effect of interaction in the evolution of the Universe. This kind of interaction term has been studied in ref. [Cai and Wang, 2005].

5.2 Field Equations and Solutions

The metric of a spatially flat homogeneous and isotropic universe in FRW model is considered in eq. (1.7) The Einstein field equations and energy conservation equation are given by in equations (1.12) and (1.13) and (1.20).

Now, we extend MCG with equation of state (1.60) such that A and B are positive function of the cosmological scale factor 'a' (i.e., $A = A(a), B = B(a)$). Then equation (1.60) reduces to,

$$p = A(a)\rho - \frac{B(a)}{\rho^{\alpha}} \quad \text{with} \quad 0 \leq \alpha \leq 1 \tag{5.1}$$

As we can see this is an inhomogeneous EOS [Brevik etal, 2007] where the pressure is a function of the energy density ρ and the scale factor $a(t)$. Also if $\rho = \left(\frac{B(a)}{A(a)}\right)^{\frac{1}{1+\alpha}}$, this model reduces to dust model, pressure being zero.

Now, assume $A(a)$ and $B(a)$ to be of the form

$$A(a) = A_0 a^{-n} \tag{5.2}$$

and

$$B(a) = B_0 a^{-m} \tag{5.3}$$

where A_0, B_0, n and m are positive constants. If $n = m = 0$, we get back the modified Chaplygin gas [Debnath etal, 2004] and if $n = 0$, we get back variable modified Chaplygin gas (VMCG) model [Debnath, 2007]. Using equations (1.20), (5.1), (5.2) and (5.3), we get the solution of ρ as,

$$\rho = a^{-3} e^{\frac{3A_0 a^{-n}}{n}} \left[C_0 + \frac{B_0}{A_0} \left(\frac{3A_0(1+\alpha)}{n} \right)^{\frac{3(1+\alpha)+n-m}{n}} \Gamma(\frac{m-3(1+\alpha)}{n}, \frac{3A_0(1+\alpha)}{n} a^{-n}) \right]^{\frac{1}{1+\alpha}} \tag{5.4}$$

where $\Gamma(a, x)$ is the upper incomplete gamma function and C_0 is an integration constant .

Now, considering the equation of state

$$\omega_{eff} = \frac{p}{\rho}$$

for this fluid, we have,

$$\omega_{eff} = A_0 a^{-n} - B_0 a^{-\zeta} e^{-\frac{3A_0(1+\alpha)a^{-n}}{n}} \left[C_0 + \left(\frac{3A_0(1+\alpha)}{n} \right)^{\frac{n-\zeta}{n}} \frac{B_0}{A_0} \Gamma(\frac{\zeta}{n}, \frac{3A_0(1+\alpha)a^{-n}}{n}) \right] \tag{5.5}$$

where $\zeta = m - 3(1+\alpha)$.

For small values of the scale factor $a(t)$, ρ is very large and

$$p = A\rho - \frac{B}{\rho^{\alpha}} \rightarrow A\rho$$

where $A = A_0 a^{-n}$ is a function of a, so that for small scale factor we have very large pressure and energy densities. Therefore initially

$$\frac{p}{\rho} = \omega_{eff} = A^* a^{-n} \leq 1$$

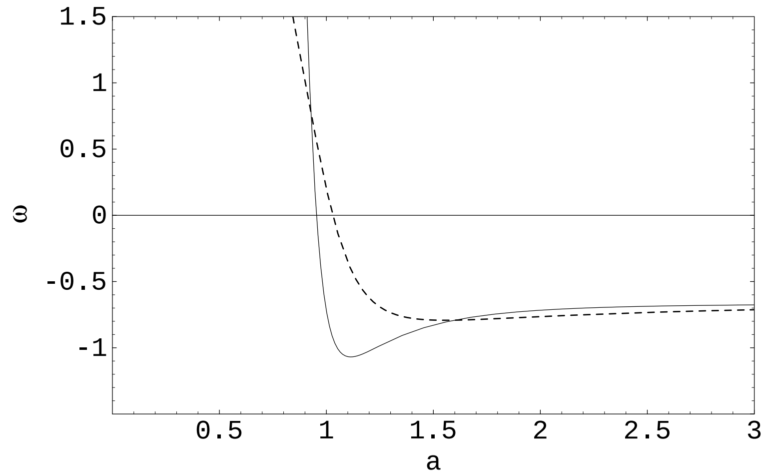

Fig 5.1: The variation of ω_{eff} is shown against $a(t)$ for $A_0 = 1, B_0 = 10, \alpha = 1, m = 2, C_0 = 1$ and $n = 3$ (for dotted line), $n = 10$ (for the dark line).

where A^* is a constant,

$$A^* = A_0.$$

If $a = A_0^{\frac{1}{n}}$, the Universe starts from stiff perfect fluid, and if $a = 3A^{*\frac{1}{n}}$, the Universe starts from radiation era.

Also for large values of the scale factor

$$p = A\rho - \frac{B}{\rho^\alpha} \to -\frac{B}{\rho^\alpha}.$$

If

$$\zeta = m - 3(1+\alpha) < 0$$

(as we know that upper incomplete Gamma function $\Gamma(a, x)$ exists for $a < 0$), the second term dominates and hence $\omega_{eff} \to -B^* a^{-\zeta}$, where

$$B^* = B_0 \lim_{a\to\infty} e^{-\frac{3A_0(1+\alpha)a^{-n}}{n}} \left[C_0 + \left(\frac{3A_0(1+\alpha)}{n}\right)^{\frac{n-\zeta}{n}} \frac{B_0}{A_0} \Gamma(\frac{\zeta}{n}, \frac{3A_0(1+\alpha)a^{-n}}{n}) \right]^{-1}$$

($\lim_{a\to\infty} e^{-\frac{3A_0(1+\alpha)a^{-n}}{n}} \to 1$ and $\lim_{a\to\infty}\Gamma(\frac{\zeta}{n}, \frac{3A_0(1+\alpha)a^{-n}}{n}) \to$large value, for $\zeta < 0$). This will represent dark energy if $a > \left(\frac{1}{3B^*}\right)^{\frac{1}{3(1+\alpha)-m}}$, ΛCDM if $a = \left(\frac{1}{B^*}\right)^{\frac{1}{3(1+\alpha)-m}}$ and phantom dark energy if $a > \left(\frac{1}{B^*}\right)^{\frac{1}{3(1+\alpha)-m}}$. Therefore we can explain the evolution of the Universe

till the phantom era depending on the various values of the parameters. We have shown a graphical representation of ω_{eff} in fig 5.1 for different values of the parameters. We can see from fig 5.1 that ω_{eff} starting from a large values decreases with a crosses $\omega = -1$ for some choices of the parameters.

5.3 Statefinder Diagnostics

Now we analyse our model using statefinder parameters given by equation (1.113) to investigate the validation of the model.

For this model

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{1}{3}\rho \tag{5.6}$$

and

$$q = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} + \frac{3}{2}\frac{p}{\rho} \tag{5.7}$$

So from equation (1.113) we get

$$r = 1 + \frac{9}{2}\left(1 + \frac{p}{\rho}\right)\frac{\partial p}{\partial \rho} - \frac{3}{2}\frac{a}{\rho}\frac{\partial p}{\partial a}\ , \qquad s = \frac{2(r-1)}{9\left(\frac{p}{\rho}\right)} \tag{5.8}$$

so that, solving we get,

$$r = 1 + \frac{9}{2}(1+y)(A_0 a^{-n} + \alpha B_0 a^{-m} x) + \frac{3}{2}(nA_0 a^{-n} - mB_0 a^{-m} x)\ , \qquad s = \frac{2(r-1)}{9y} \tag{5.9}$$

where, $y = \frac{p}{\rho} = A_0 a^{-n} - B_0 a^{-m} x$ and $x = \rho^{-(1+\alpha)}$, ρ is given by equation (5.4).

We have plotted the $\{r, s\}$ parameters in figure 5.2 normalizing the parameters and varying the scale factor $a(t)$. We can see that the model starts from radiation era. Then

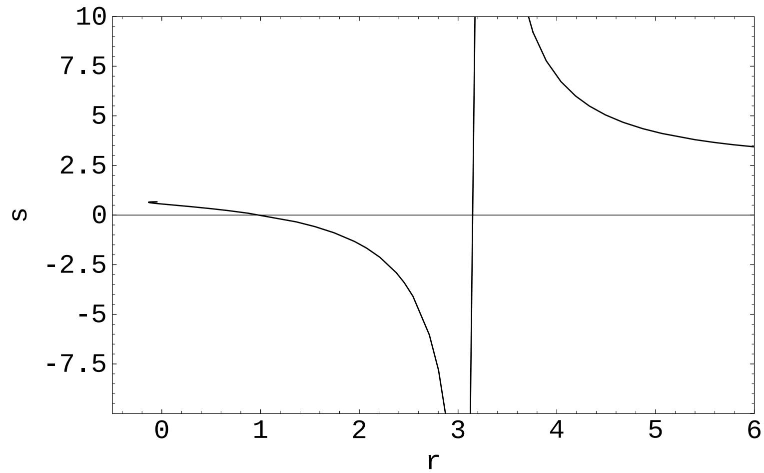

Fig 5.2: The variation of s is plotted against r for $A_0 = 1, B_0 = 1, \alpha = \frac{1}{2}, m = 3, n = 2, C_0 = 1$.

we have a discontinuity at the dust era (for radiation era: $s > 0$ and $r > 1$; dust era: $r > 1$ and $s \to \pm\infty$; ΛCDM: $r = 1$, $s = 0$; phantom: $r < 1$). The model reaches ΛCDM at $r = 1$, $s = 0$ and then crosses ΛCDM to represent phantom dark energy. This model represents the phantom dark energy, whereas, Modified Chaplygin Gas can explain the evolution of the Universe from radiation to ΛCDM and Variable Modified Chaplygin gas describes the evolution of the Universe from radiation to quiessence model.

5.4 New modified Chaplygin gas and interacting scalar field

Now we consider model of interaction between scalar field and the new modified Chaplygin Gas model, through a phenomenological interaction term. Keeping into consideration the fact that the Supernovae and CMB data determines that decay rate should be proportional to the present value of the Hubble parameter [Berger and Shojaei, 2006], we have chosen the interaction term likewise. This interaction term describes the energy flow between the two fluids. We have considered a scalar field to couple with the New Modified Chaplygin gas given by EOS (5.1), (5.2) and (5.3).

Therefore now the conservation equation becomes equation (4.4). For the interacting

model, the equations of motion of the the new fluid and scalar field read,

$$\dot{\rho} + 3H(\rho + p) = -3H\rho\delta \tag{5.10}$$

and

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 3H\rho\delta \tag{5.11}$$

(δ is a constant).

Where the total energy density and pressure of the universe are given by,

$$\rho_{tot} = \rho + \rho_\phi \tag{5.12}$$

and

$$p_{tot} = p + p_\phi \tag{5.13}$$

where, ρ and p are the energy density and pressure of the extended modified Chaplygin gas model given by equations (5.1), (5.2), (5.3), (5.4) and ρ_ϕ and p_ϕ are the energy density and pressure due to the scalar field given by,

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi) \tag{5.14}$$

and

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) \tag{5.15}$$

where, $V(\phi)$ is the relevant potential for the scalar field ϕ.

Thus from the field equations (4.2) and (4.3) and the conservation equation (4.3), we get the solution for ρ as

$$\rho = a^{-3(1+\delta)} e^{\frac{3A_0 a^{-n}}{n}} \left[C_0 + \frac{B_0}{A_0} \left(\frac{3A_0(1+\alpha)}{n} \right)^{\frac{3(1+\alpha)(1+\delta)+n-m}{n}} \right.$$

$$\left. \Gamma\left(\frac{m-3(1+\alpha)(1+\delta)}{n}, \frac{3A_0(1+\alpha)}{n} a^{-n}\right) \right]^{\frac{1}{1+\alpha}} \tag{5.16}$$

where C_0 is an integration constant.

Further substitution in the above equations give,

$$V(\phi) = 3H^2 + \dot{H} + \frac{p-\rho}{2} \tag{5.17}$$

To get an explicit form of the energy density and the potential corresponding to the scalar field we consider a power law expansion of the scale factor $a(t)$ as,

$$a = t^{\beta} \tag{5.18}$$

so that, for $\beta > 1$ we get accelerated expansion of the Universe thus satisfying the observational constrains. If $\beta = 1$ or $\beta < 1$ we get constant and decelerated expansion respectively.

Using equations (5.6), (5.12) and (5.18), we get,

$$\rho_\phi = \frac{3\beta^2}{t^2} - \rho \tag{5.19}$$

where ρ is given by equation (5.16).

Also the potential takes the form,

$$V = \frac{3n^2 - n}{t^2} + \frac{p-\rho}{2} \tag{5.20}$$

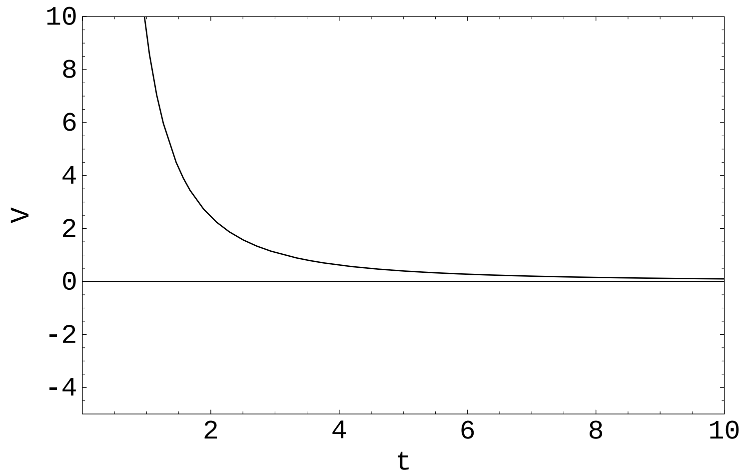

Fig 5.3: The variation of V is plotted against t for $A_0 = 1, B_0 = 1, \alpha = \frac{1}{2}, m = 3, n = 2, C_0 = 0$.

The graphical representation of V against time is shown in figure 5.3 normalizing the parameters. We see that the potential decays with time.

5.5 Discussion

Here we present a new variable modified Chaplygin gas model which is an unified version of the dark matter and the dark energy of the Universe. It behaves like dark matter at the initial stage and later it explains the dark energy of the Universe. Unlike the Generalized or Modified Chaplygin gas model, it can explain the evolution of the Universe at phantom era depending on the parameters. Also we have calculated the $\{r, s\}$ parameters corresponding to this model. Normalizing the parameters such that $m-3(1+\alpha) < 0$, show the diagrammatical representation of $\{r, s\}$ for our model (in Fig. 5.2), varying the scale factor. We see that starting from the radiation era it crosses $\omega = -1$ and extends till phantom era. Also we can see that the deceleration parameter starting from a positive point becomes negative, indicating deceleration initially and acceleration at later times. Again we have considered an interaction of this fluid with that of scalar field by introducing a phenomenological coupling term, so that there is a flow of energy between the field and the fluid which decays with time, as in the initial stage the fluid behaves more like dark matter and the field that of dark energy, whereas in the later

stage both explain the dark energy present in the Universe. In Fig. 5.3, we have shown the nature of the potential due to the scalar field considering a power law expansion of the Universe to keep the recent observational support of cosmic acceleration, and we see that the potential decays with time.

Chapter 6

Dynamics of Tachyonic Field in Presence of Perfect Fluid

6.1 Prelude

Recently tachyonic field with Lagrangian $\mathcal{L} = -V(T)\sqrt{1 + g^{\mu\nu}\ \partial_\mu T \partial_\nu T}$ [Sen, 2002] has gained a lot of importance as dark energy model. The energy-momentum tensor of the tachyonic field can be seen as a combination of two fluids, dust with pressure zero and a cosmological constant with $p = -\rho$, thus generating enough negative pressure such as to drive acceleration. Also the tachyonic field has a potential which has an unstable maximum at the origin and decays to almost zero as the field goes to infinity. Depending on various forms of this potential following this asymptotic behaviour a lot of works have been carried out on tachyonic dark energy [Bagla etal, 2003; Copeland etal, 2005, 2006], tachyonic dark matter [Padmanabhan, 2002; Das etal, 2005] and inflationary models [Sami, 2003]. Recently, interacting tachyonic-dark matter model has also been studied [Herrera etal, 2004].

In this chapter, we consider a model which comprises of a two component mixture. Firstly we consider a mixture of barotropic fluid with tachyonic field without any interaction between them, so that both of them retain their properties separately. Then we consider an energy flow between them by introducing an interaction term which is proportional to the product of the Hubble parameter and the density of the barotropic fluid. We show that the energy flow being considerably high at the beginning falls down

noticeably with the evolution of the Universe indicating a more stable situation. Also in both the cases we find the exact solutions for the tachyonic field and the tachyonic potential and show that the tachyonic potential follows the asymptotic behaviour discussed above. Here the tachyonic field behaves as the dark energy component whereas the dust acts as the cold dark matter. Next we consider tachyonic dark matter, the Generalized Chaplygin Gas (GCG) being the dark energy component. GCG, identified by the equation of state (EOS) (1.53) has been considered as a suitable dark energy model by several authors [Bento etal, 2002; Gorini etal, 2003]. Here we consider the mixture of GCG with tachyonic dark matter. Later we have also considered an interaction between these two fluids by introducing a coupling term which is proportional to the product of Hubble constant and the energy density of the GCG. The coupling function decays with time indicating a strong energy flow at the initial period and weak interaction at later stage implying a stable situation. Here we have found the exact solution of the tachyonic potential. To keep the observational support of recent acceleration we have considered a particular form of evolution of the Universe here as

$$a = t^n \tag{6.1}$$

such that the deceleration parameter reads $q = -\frac{a\ddot{a}}{\dot{a}^2} = -(1 - \frac{1}{n})$, where a is the scale factor. Hence for $n > 1$ we always get an accelerated expansion and for $n = 1$ we get a constant expansion of the Universe. This kind of recipe has been studied in ref. [Padmanabhan, 2002].

6.2 Field Equations

The action for the homogeneous tachyon condensate of string theory in a gravitational background is given by,

$$S = \int \sqrt{-g}\, d^4x \left[\frac{\mathcal{R}}{16\pi G} + \mathcal{L}_{tach}\right] \tag{6.2}$$

where $\mathcal{L}$ is the Lagrangian density given by equation (1.66), where T is the tachyonic field, $V(T)$ is the tachyonic potential and $\mathcal{R}$ is the Ricci Scalar. The energy-momentum tensor for the tachyonic field is,

$$\begin{aligned} T_{\mu\nu} = -\tfrac{2\delta S}{\sqrt{-g}\ \delta g^{\mu\nu}} &= -V(T)\sqrt{1+g^{\mu\nu}\partial_\mu T\partial_\nu T}g^{\mu\nu} + V(T)\tfrac{\partial_\mu T\partial_\nu T}{\sqrt{1+g^{\mu\nu}\partial_\mu T\partial_\nu T}} \\ &= p_T\ g_{\mu\nu} + (p_T+\rho_T)u_\mu u_\nu \end{aligned} \tag{6.3}$$

where the velocity u_μ is :

$$u_\mu = -\frac{\partial_\mu T}{\sqrt{-g^{\mu\nu}\partial_\mu T\partial_\nu T}} \tag{6.4}$$

with $u^\nu u_\nu = -1$.

The energy density ρ_T and the pressure p_T of the tachyonic field therefore are given by (1.71) and (1.72) respectively. Hence the EOS parameter of the tachyonic field becomes (1.73) and (1.74), which represents pure Chaplygin gas if $V(T)$ is constant.

Now the metric of a spatially flat isotropic and homogeneous Universe in FRW model is presented by equation (1.7). The Einstein field equations are (4.2) and (4.3), where, ρ_{tot} and p_{tot} are the total energy density and the pressure of the Universe. The energy conservation equation is given by equation (4.4).

6.3 Tachyonic Dark Energy in presence of Barotropic Fluid

Now we consider a two fluid model consisting of tachyonic field and barotropic fluid. The EOS of the barotropic fluid is given by,

$$p_b = \omega_b \rho_b \tag{6.5}$$

where p_b and ρ_b are the pressure and energy density of the barotropic fluid. Hence the total energy density and pressure are respectively given by,

$$\rho_{tot} = \rho_b + \rho_T \tag{6.6}$$

and

$$p_{tot} = p_b + p_T \tag{6.7}$$

6.3.1 Without Interaction

First we consider that the two fluids do not interact with each other so that they are conserved separately. Therefore, the conservation equation (4.4) reduces to,

$$\dot{\rho}_T + 3\frac{\dot{a}}{a}(\rho_T + p_T) = 0 \tag{6.8}$$

and

$$\dot{\rho}_b + 3\frac{\dot{a}}{a}(\rho_b + p_b) = 0 \tag{6.9}$$

Equation (6.9) together with equation (6.5) gives,

$$\rho_b = \rho_0 \; a^{-3(1+\omega_b)} \tag{6.10}$$

Now, we consider a power law expansion of the scale factor $a(t)$ given by equation (6.1).

Using (6.1), equation (6.10) reduces to,

$$\rho_b = \rho_0 \; t^{-3n(1+\omega_b)} \tag{6.11}$$

Also the energy density corresponding to the tachyonic field becomes,

$$\rho_T = \frac{1}{t^2}\left[3n^2 - \rho_0\ t^{-3n(1+\omega_b)+2}\right] \tag{6.12}$$

Solving the equations the tachyonic field is obtained as,

$$T = \sqrt{1+\omega_b}t\ Appell\ F_1\left[\frac{1}{3(1+\omega_b)n-2}, \frac{1}{2}, -\frac{1}{2}, 1+\frac{1}{3(1+\omega_b)n-2},\right.$$

$$\left.\frac{3n^2}{\rho_0}t^{3(1+\omega_b)n-2}, \frac{2n}{\rho_0(1+\omega_b)}t^{3(1+\omega_b)n-2}\right] \tag{6.13}$$

where, $Appell\ F1[a, b_1, b_2, c, x, y]$ is the Appell Hypergeometric function of two variables x and y.

Also the potential will be of the form,

$$V(T) = \sqrt{\frac{3n^2}{t^2} - \rho_0\ t^{-3n(1+\omega_b)}}\sqrt{\frac{3n^2}{t^2} - \frac{2n}{t^2} + \omega_b\rho_0\ t^{-3n(1+\omega_b)}} \tag{6.14}$$

We can show the graphical representation of the potential against time in figure 6.1. We can see that $V \to 0$ with time, thus retaining the original property of the tachyon potential.

6.3.2 With Interaction

Now we consider an interaction between the tachyonic field and the barotropic fluid by introducing a phenomenological coupling function which is a product of the Hubble parameter and the energy density of the barotropic fluid. Thus there is an energy flow between the two fluids.

Now the equations of motion corresponding to the tachyonic field and the barotropic fluid are respectively,

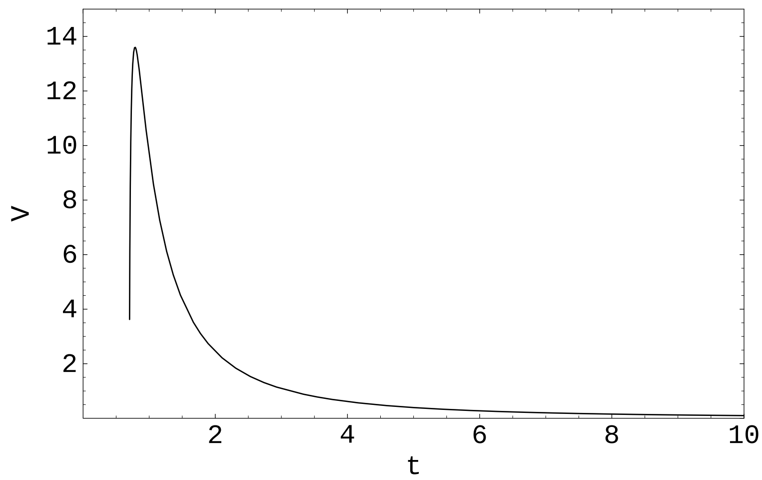

Fig 6.1: The variation of V is plotted against t for $n = 2, \rho_0 = 1, \omega_b = \frac{1}{2}$.

$$\dot{\rho}_T + 3\frac{\dot{a}}{a}(\rho_T + p_T) = -3H\delta\rho_b \tag{6.15}$$

and

$$\dot{\rho}_b + 3\frac{\dot{a}}{a}(\rho_b + p_b) = 3H\delta\rho_b \tag{6.16}$$

where δ is a coupling constant.

Solving equation (6.16) with the help of equation (6.5), we get,

$$\rho_b = \rho_0 \ a^{-3(1+\omega_b-\delta)} \tag{6.17}$$

Considering the power law expansion (6.1), we get

$$\rho_b = \rho_0 \ t^{-3n(1+\omega_b-\delta)} \tag{6.18}$$

Equation (6.2) and (6.18) give,

$$\rho_T = \frac{3n^2}{t^2} - \rho_0 \ t^{-3n(1+\omega_b-\delta)} \tag{6.19}$$

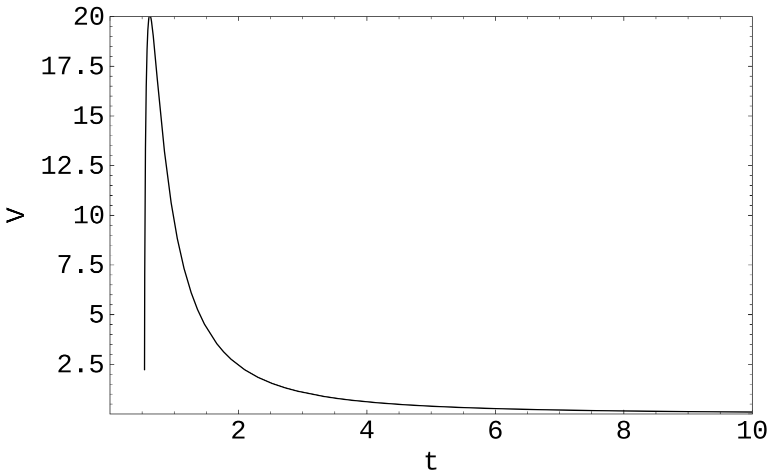

Fig 6.2: The variation of V is plotted against t for $n = 2, \rho_0 = 1, \omega_b = \frac{1}{2}, \delta = \frac{1}{2}$.

Solving the equations the tachyonic field is obtained as,

$$T = \sqrt{1+\omega_b}t \ Appell \ F_1 \left[\frac{1}{3(1+\omega_b-\delta)n-2}, \frac{1}{2}, -\frac{1}{2},\right.$$

$$\left. 1 + \frac{1}{3(1+\omega_b-\delta)n-2}, \frac{3n^2}{\rho_0} t^{3(1+\omega_b-\delta)n-2}, \frac{2n}{\rho_0(1+\omega_b)} t^{3(1+\omega_b-\delta)n-2}\right] \tag{6.20}$$

Also the potential will be of the form,

$$V(T) = \sqrt{\frac{3n^2}{t^2} - \rho_0 \ t^{-3n(1+\omega_b-\delta)}} \sqrt{\frac{3n^2}{t^2} - \frac{2n}{t^2} + \omega_b \rho_0 \ t^{-3n(1+\omega_b-\delta)}} \tag{6.21}$$

In this case also $V \to 0$ with time as shown in the graphical representation of V in figure 6.2.

6.4 Tachyonic Dark Matter in presence of GCG

Now we consider a two fluid model consisting of tachyonic field and GCG. The EOS of GCG is given by,

$$p_{ch} = -B/\rho_{ch}^{\alpha} \qquad\qquad 0 \le \alpha \le 1, B > 0. \tag{6.22}$$

where p_{ch} and ρ_{ch} are the pressure and energy density of GCG. Hence the total energy density and pressure are respectively given by,

$$\rho_{tot} = \rho_{ch} + \rho_T \tag{6.23}$$

and

$$p_{tot} = p_{ch} + p_T \tag{6.24}$$

6.4.1 Without Interaction

First we consider that the two fluids do not interact with each other so that they are conserved separately. Therefore, the conservation equation (4.4) reduces to,

$$\dot{\rho}_T + 3\frac{\dot{a}}{a}(\rho_T + p_T) = 0 \tag{6.25}$$

and

$$\dot{\rho}_{ch} + 3\frac{\dot{a}}{a}(\rho_{ch} + p_{ch}) = 0 \tag{6.26}$$

Equation (6.26) together with equation (6.22) give,

$$\rho_{ch} = \left[B + \frac{\rho_{00}}{a^{3(1+\alpha)}}\right]^{\frac{1}{(1+\alpha)}} \tag{6.27}$$

Using (6.1), equation (6.27) reduces to

$$\rho_{ch} = \left[B + \rho_{00}t^{-3n(1+\alpha)}\right]^{\frac{1}{(1+\alpha)}} \tag{6.28}$$

Hence the energy density of the tachyonic fluid is,

$$\rho_T = \frac{3n^2}{t^2} - \left[B + \rho_{00}t^{-3n(1+\alpha)}\right]^{\frac{1}{(1+\alpha)}} \tag{6.29}$$

Solving the equations the tachyonic field and the tachyonic potential are obtained as,

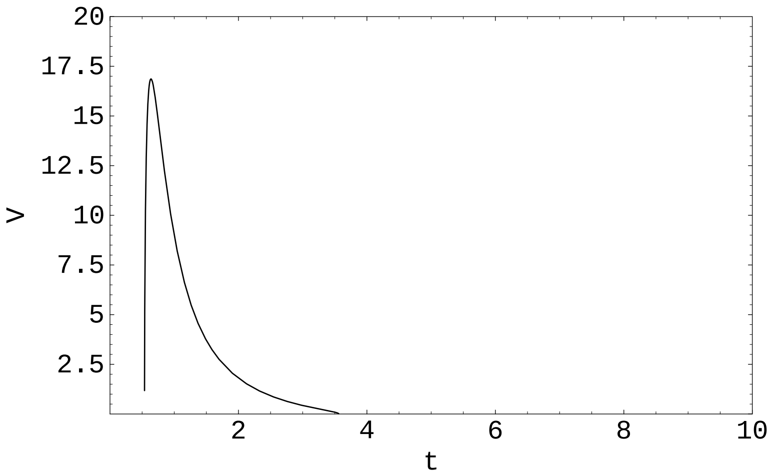

Fig 6.3: The variation of V is plotted against t for $B=\frac{1}{2}, n=2, \rho_{00}=1, \alpha=\frac{1}{2}$.

$$T=\int\sqrt{\frac{\frac{2n}{t^2}-\rho_{00}t^{-3n(1+\alpha)}\left[B+\rho_{00}t^{-3n(1+\alpha)}\right]^{-\frac{\alpha}{(1+\alpha)}}}{\frac{3n^2}{t^2}-\left[B+\rho_{00}t^{-3n(1+\alpha)}\right]^{\frac{1}{(1+\alpha)}}}}dt \tag{6.30}$$

Also the potential will be of the form,

$$V(T)=\sqrt{\frac{3n^2}{t^2}-\left[B+\rho_{00}t^{-3n(1+\alpha)}\right]^{\frac{1}{(1+\alpha)}}}\sqrt{\frac{3n^2}{t^2}-\frac{2n}{t^2}-B\left[B+\rho_{00}t^{-3n(1+\alpha)}\right]^{-\frac{\alpha}{(1+\alpha)}}} \tag{6.31}$$

Like the mixture of tachyonic fluid with barotropic fluid in this case also the potential V starting from a low value increases largely and then decreases to 0 with time as shown in figure 6.3.

6.4.2 With Interaction

Now we consider an interaction between the tachyonic fluid and GCG by phenomenologically introducing an interaction term as a product of the Hubble parameter and the energy density of the Chaplygin gas. Thus there is an energy flow between the two fluids.

Now the equations of motion corresponding to the tachyonic field and GCG are respectively,

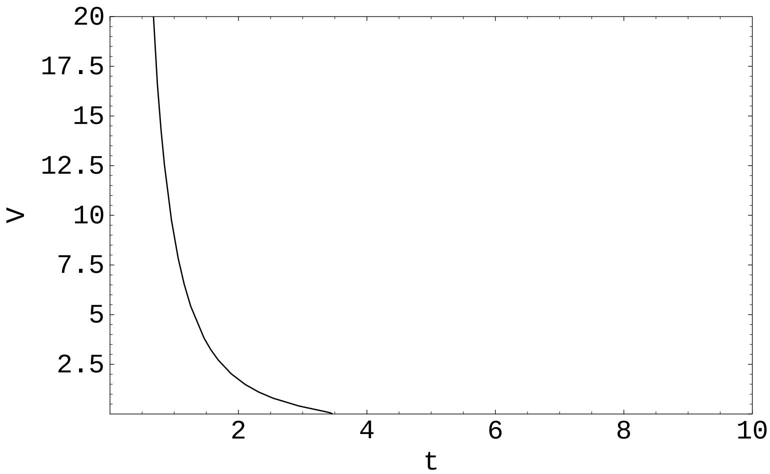

Fig 6.4: The variation of V is plotted against t for $B = \frac{1}{2}, n = 2, \rho_{00} = 1, \alpha = \frac{1}{2}, \epsilon = \frac{1}{2}$.

$$\dot{\rho}_T + 3\frac{\dot{a}}{a}(\rho_T + p_T) = -3H\epsilon\rho_{ch} \tag{6.32}$$

and

$$\dot{\rho}_{ch} + 3\frac{\dot{a}}{a}(\rho_{ch} + p_{ch}) = 3H\epsilon\rho_{ch} \tag{6.33}$$

where ϵ is a coupling constant.

Solving equation (6.33) with the help of equation (6.22) and (6.1), we get,

$$\rho_{ch} = \left[\frac{B}{1-\epsilon} + \rho_{00} t^{-3n(1+\alpha)(1-\epsilon)}\right]^{\frac{1}{(1+\alpha)}} \tag{6.34}$$

Also the energy density of the tachyonic field will be read as,

$$\rho_T = \frac{3n^2}{t^2} - \left[\frac{B}{1-\epsilon} + \rho_{00} t^{-3n(1+\alpha)(1-\epsilon)}\right]^{\frac{1}{(1+\alpha)}} \tag{6.35}$$

Solving the equations the tachyonic field is obtained as,

$$T = \int \sqrt{\frac{\frac{2n}{t^2} - \left[\frac{B\epsilon}{1-\epsilon} + \rho_{00} t^{-3n(1+\alpha)(1-\epsilon)}\right]\left[\frac{B}{1-\epsilon} + \rho_{00} t^{-3n(1+\alpha)(1-\epsilon)}\right]^{-\frac{\alpha}{(1+\alpha)}}}{\frac{3n^2}{t^2} - \left[\frac{B}{1-\epsilon} + \rho_{00} t^{-3n(1+\alpha)(1-\epsilon)}\right]^{\frac{1}{(1+\alpha)}}}}\, dt \tag{6.36}$$

Also the potential will be of the form,

$$V(T) = \sqrt{\frac{3n^2}{t^2} - \left[\frac{B}{1-\epsilon} + \rho_{00} t^{-3n(1+\alpha)(1-\epsilon)}\right]^{\frac{1}{(1+\alpha)}}}$$
$$\sqrt{\frac{3n^2}{t^2} - \frac{2n}{t^2} - B\left[\frac{B}{1-\epsilon} + \rho_{00} t^{-3n(1+\alpha)(1-\epsilon)}\right]^{-\frac{\alpha}{(1+\alpha)}}} \tag{6.37}$$

In this case the potential starting from a large value tends to 0 (figure 6.4).

6.5 Discussion

We have considered the flat FRW Universe driven by a mixture of tachyonic field and a perfect fluid. We have considered barotropic fluid and Chaplygin gas for this purpose. We have presented accelerating expansion of our Universe due to interaction/without interaction of the mixture of these fluids. We have found the exact solution of the density and potential by considering a power law expansion of the scale factor. We show that these potentials represent the same decaying nature regardless the interaction between the concerned fluids. Since we have considered a power law expansion of the scale factor of the form $a = t^n$, we see that for the present acceleration of the Universe to support the observational data we need $n > 1$. Now we consider the interaction terms between these fluids. For the mixture of barotropic fluid with tachyonic fluid, we see that the interaction term reduces the potential. Also for the mixture of GCG with tachyonic fluid the interaction parameter ϵ satisfying $0 < \epsilon < 1$ so that equation (6.34) exists for smaller values of t. In this case also the interaction reduces the potential. Also if we consider only tachyonic fluid with the power law expansion, we see that the potential (which is obtained to be $V = \frac{3n^2}{t^2}\sqrt{1 - \frac{2}{3n}}$) is greater than that we get in mixtures. Also the potentials differ in the two cases we have considered. For the mixture with GCG the potential decreases faster than that in case of mixture with barotropic fluid.

Chapter 7

Interacting Model of Inhomogeneous EOS and Scalar Field

7.1 Prelude

Presently we live in an epoch where the densities of the dark energy and the dark matter are comparable. It becomes difficult to solve this coincidence problem without a suitable interaction. Generally interacting dark energy models are studied to explain the cosmic coincidence problem [Cai and Wang, 2005]. Also the transition from matter domination to dark energy domination can be explained through an appropriate energy exchange rate. Therefore, to obtain a suitable evolution of the Universe an interaction is assumed and the decay rate should be proportional to the present value of the Hubble parameter for good fit to the expansion history of the Universe as determined by the Supernovae and CMB data [Berger etal, 2006]. A variety of interacting dark energy models have been proposed and studied for this purpose [Zimdahl, 2005; Hu and Ling, 2006].

Although a lot of models have been proposed to examine the nature of the dark energy, it is not known what is the fundamental nature of the dark energy. Usually models mentioned above are considered for producing the present day acceleration. Also there is modified gravity theories where the EOS depends on geometry, such as Hubble parameter. It is therefore interesting to investigate models that involve EOS different from the usual ones, and whether these EOS is able to give rise to cosmological models meeting the present day dark energy problem. In this chapter, we consider model of

interaction between scalar field and an ideal fluid with inhomogeneous equation of state (EOS), through a phenomenological interaction which describes the energy flow between them. Ideal fluids with inhomogeneous EOS were introduced in [Nojiri etal, 2005, 2006; Elizalde etal, 2005]. Here we have considered two exotic kind of equation of states which were studied in [Brevik etal, 2004, 2007; Capozziello, 2006] with a linear inhomogeneous EOS. Here we take the inhomogeneous EOS to be in polynomial form to generalize the case. Also, the ideal fluid present here behaves more like dark matter dominated by the scalar field so that the total energy density and pressure of the Universe decreases with time. Also the potential corresponding to the scalar field shows a decaying nature. Here we have considered a power law expansion of the scale factor, so that we always get a non-decelerated expansion of the Universe for the power being greater than or equal to unity. We have solved the energy densities of both the scalar field and ideal fluid and the potential of the scalar field. Also a decaying nature of the interaction parameter is shown.

7.2 Field Equations

The metric of a spatially flat isotropic and homogeneous Universe in FRW model is given by equation (1.7). The Einstein field equations and energy conservation equation are in equations (4.2), (4.3) and (4.4). Here, ρ_{tot} and p_{tot} are the total energy density and the pressure of the Universe, given by,

$$\rho_{tot} = \rho_\phi + \rho_d \tag{7.1}$$

and

$$p_{tot} = p_\phi + p_d \tag{7.2}$$

with ρ_ϕ and p_ϕ are respectively the energy density and pressure due to the scalar field given by equations (5.14) and (5.15) respectively. Also, ρ_d and p_d are the energy density

and the pressure corresponding to the ideal fluid with an inhomogeneous EOS,

$$p_d = \omega(t)\rho_d + \omega_1 f(H,t) \tag{7.3}$$

where, $\omega(t)$ is a function of t and $f(H,t)$ is a function of H and t (H is the Hubble parameter $= \frac{\dot{a}}{a}$).

Now we consider the scalar field interacting with the ideal fluid with inhomogeneous EOS through an energy exchange between them. The equations of motion of the scalar field and the ideal fluid can be written as,

$$\dot{\rho_d} + 3H(\rho_d + p_d) = -3H\rho_d\delta \tag{7.4}$$

and

$$\dot{\rho_\phi} + 3H(\rho_\phi + p_\phi) = 3H\rho_d\delta \tag{7.5}$$

where δ is coupling constant.

7.2.1 Case I: Model with EOS in power law form

Taking into account the recent cosmological considerations of variations of fundamental constants, one may start from the case that the pressure depends on the time t [Brevik etal, 2004]. Unlike the EOS studied in [Brevik etal, 2007] where the parameters involved in EOS are linear in t, we consider rather a polynomial form. First, we choose the EOS of the ideal fluid to be,

$$p_d = a_1 t^{-\alpha}\rho_d - ct^{-\beta} \tag{7.6}$$

where, a_1, c, α, β are constants.

Here, we see that initially the pressure is very large and as time increases pressure falls down, which is very much compatible with the recent observational data.

We consider a Universe with power law expansion given by equation (6.1), so as to get a non-decelerated expansion for $n \geq 1$, as the deceleration parameter reduces to $q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1-n}{n} < 0$.

Now equation (7.4) together with (7.6) and (6.1) gives the solution for ρ_d to be,

$$\rho_d = t^{-3n(1+\delta)} e^{\frac{3na_1 t^{-\alpha}}{\alpha}} \left(\frac{3na_1}{\alpha}\right)^{\frac{3n(1+\delta)+\alpha-\beta}{\alpha}} \frac{c}{a_1} \Gamma(\frac{\beta - 3n(1+\delta)}{\alpha}, \frac{3na_1 t^{-\alpha}}{\alpha}) \tag{7.7}$$

where, $\Gamma(a, x)$ is upper incomplete Gamma function.

Further substitution in the above equations give the solution for $\rho_\phi, \dot{\phi}^2$ and $V(\phi)$ to be,

$$\rho_\phi = 3\frac{n^2}{t^2} - \rho_d \tag{7.8}$$

$$\dot{\phi}^2 = \frac{2n}{t^2} - \left[(1 + a_1 t^{-\alpha})\rho_d - ct^{-\beta}\right] \tag{7.9}$$

so that,

$$\phi = \phi_0 + \int \sqrt{\frac{2n}{t^2} - [(1 + a_1 t^{-\alpha})\rho_d - ct^{-\beta}]}\, dt \tag{7.10}$$

and

$$V = \frac{3n^2 - n}{t^2} + \frac{(-1 + a_1 t^{-\alpha})\rho_d}{2} - \frac{ct^{-\beta}}{2} \tag{7.11}$$

Since we have considered a power law expansion of the scale factor so we can see from the above expressions that ρ_d and ρ_ϕ are decreasing functions of time so that the total

energy density as well as pressure decreases with time. The evolution of the Universe therefore can be explained without any singularity. Normalizing the parameters, we get the variation of $V(\phi)$ against ϕ in figure 7.3. Equation (7.11) shows that, for $\beta < 2$, the potential being positive initially, may not retain this as $t \to \infty$ (as the 3rd term dominates over first term and the third and second term being negative for large values); for $\beta = 2$ the potential can be positive depending on the value of $\left(3n^2 - n - \frac{c}{2}\right)$ and for, $\beta > 2$ the potential can be either positive depending on the choices of the constants, but always decreases with time. Hence β is completely arbitrary and depending on various values of β and the other constants, potential to be positive, although it is always decreasing with time. Fig 7.3 shows the nature of the potential for arbitrarily chosen values of the constants. Also if we consider $w_d = \frac{p_d}{\rho_d}, w_\phi = \frac{p_\phi}{\rho_\phi}, w_{tot} = \frac{p_{tot}}{\rho_{tot}}$, and plot them (figure 7.1) against time, we see this represents an XCDM model and therefore it makes a positive contribution to $\ddot{a}/a$.

7.2.2 Case II: Model with EOS depending on Hubble parameter

Inhomogeneous dark energy EOS coming from geometry, for example, H can yield cosmological models which can avoid shortcomings coming from coincidence problem and a fine-tuned sudden evolution of the Universe from the early phase of deceleration driven by dark matter to the present phase of acceleration driven by dark energy. Furthermore, such models allow to recover also early accelerated regimes with the meaning of inflationary behaviors [Capozziello etal, 2006]. The following model is often referred to as Increased Matter Model where the pressure depends on energy density and H. A detailed discussion of this kind of EOS can be found in ref. [Capozziello etal, 2006].

Now we choose the EOS of the ideal fluid to be,

$$p_d = A\rho_d + BH^2 \tag{7.12}$$

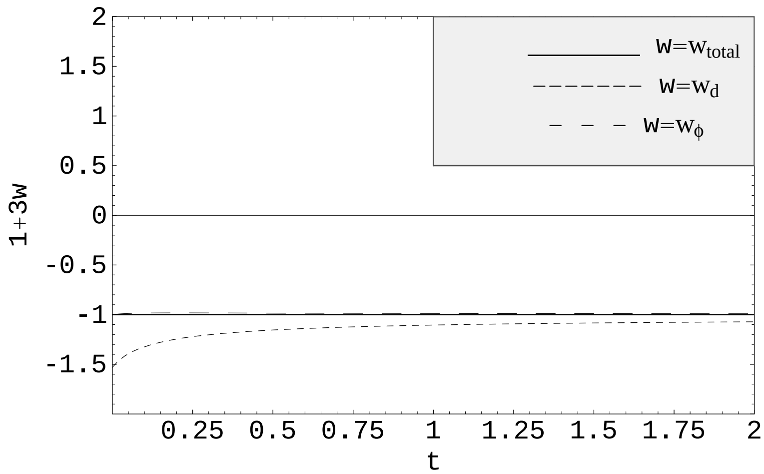

Fig 7.1: The variation of $1+3w$ is plotted where $w = w_d, w_\phi, w_{tot}$ against time, normalizing the parameters as $n = 2, \alpha = 1, \beta = 2, a_1 = .1, c = 1, \delta = .01$.

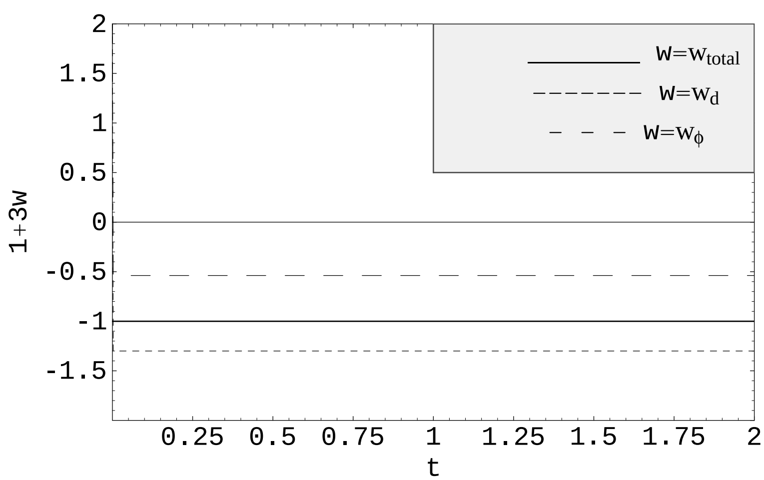

Fig 7.2: The variation of $1+3w$ is plotted where $w = w_d, w_\phi, w_{tot}$ against time, normalizing the parameters as $A = \frac{1}{3}, B = -2, \phi_0 = 1, \delta = .1, n = 2$.

where, A and B are constants.

Considering the power law expansion (6.1) and using (7.4) and (7.12), we get the solution for ρ_d to be,

$$\rho_d = C_0 t^{-3n(1+A+\delta)} - \frac{3n^3 B}{3n(1+A+\delta)-2} t^{-2} \tag{7.13}$$

Further substitution in the related equations yields the solution for $\rho_\phi, \phi, V(\phi)$ to be,

$$\rho_\phi = \frac{3n^2}{t^2} - \rho_d \tag{7.14}$$

$$\phi = \phi_0 + \frac{2}{2-K_3}\left[\sqrt{K_1 + K_2 t^{2-K_3}} - \sqrt{K_1}\sinh^{-1}\left(\sqrt{\frac{K_1}{K_2}}x\right)\right] \tag{7.15}$$

where, $x = t^{\frac{K_3}{2}-1}, K_2 = -C_0(1+A), K_1 = \frac{6n^2(1+A+\delta)-4n-3Bn^3\delta+2Bn^2}{K_3-2}, K_3 = 3n(1+A+\delta)$

and

$$V = \frac{3n^2-n}{t^2} + \frac{A-1}{2}\rho_d + \frac{Bn^2}{2t^2} \tag{7.16}$$

Equation (7.15) shows that K_1 must be positive and hence K_2 also must be positive for a valid expression. Also equation (7.13) says that C_0 must be positive, otherwise ρ_d becomes negative initially. Therefore expression of K_2 says that A must be negative, in fact, $A < -1$, such that depending on the value of B pressure can be positive or negative. Normalizing the parameters, we get the variation of V against ϕ in figure 7.4. The figure shows a decaying nature of the potential. Also if we consider $w_d = \frac{p_d}{\rho_d}, w_\phi = \frac{p_\phi}{\rho_\phi}, w_{tot} = \frac{p_{tot}}{\rho_{tot}}$, and plot them (figure 7.2) against time, like the previous case, we see this represents an XCDM model and therefore it makes a positive contribution to $\ddot{a}/a$.

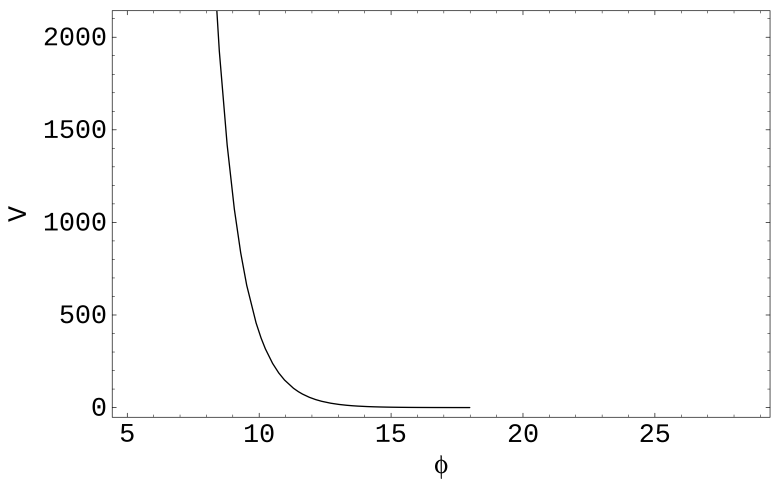

Fig 7.3: The variation of V has been plotted against ϕ normalizing the parameters as $n = 2, \alpha = 1, \beta = 2, a_1 = .1, c = 1, \delta = .01$.

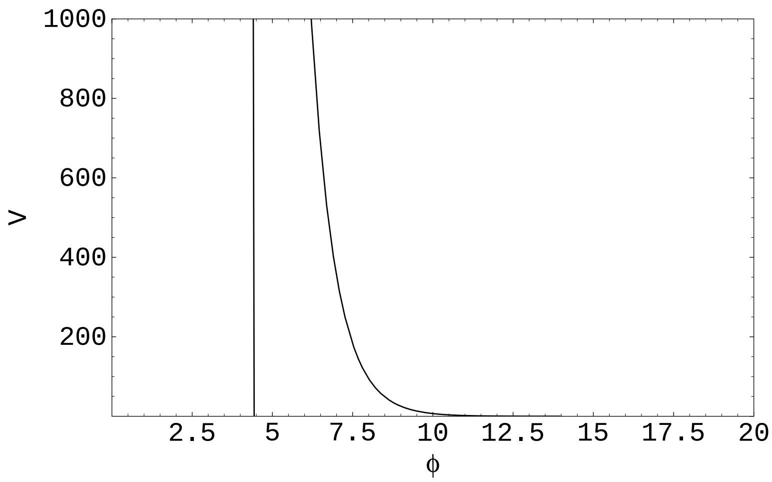

Fig 7.4: The variation of V has been plotted against ϕ normalizing the parameters as $A = \frac{1}{3}, B = -2, \phi_0 = 1, \delta = .1, n = 2$.

7.3 Discussion

In this chapter, we study a cosmological model of the Universe in which the scalar field has an interaction with an ideal fluid with inhomogeneous EOS. The interaction is introduced phenomenologically by considering term parameterized by the product of the Hubble parameter, the energy density of the ideal fluid and a coupling constant in the equations of motion of the fluid and the scalar field. This type of phenomenological interaction term has been investigated in [Cai and Wang, 2005]. This describes an energy flow between the scalar field and the ideal fluid. Also we consider a power law form of the scale factor $a(t)$ to keep the recent observational support of cosmic acceleration. For the first model putting $c = 0, \alpha = 0$ we get the results for barotropic fluid. Here for α and β to be positive, the ideal fluid and the scalar field behave as dark energy. Also we see that the interaction term decreases with time showing strong interaction at the earlier stage and weak interaction later. Also the potential corresponding to the scalar field is positive and shows a decaying nature. In the second model where p_d is a function of ρ_d and the Hubble parameter H, we see that the energy density and the pressure of the ideal fluid and that of the scalar field always decreases with time. From figures 7.3 and 7.4, we see that, the potential function V decreases for both decelerating $(n < 1)$ and accelerating phase $(n > 1)$. Also from the values of density and pressure terms, it can be shown that the individual fluids and their mixtures satisfy strong energy condition for $n < 1$ and violate for $n > 1$. A detailed discussion of the potential of a scalar field can be found in ref. [Cardenas etal, 2004]. We see that the coupling parameter shows a decaying nature in both the cases implying strong interaction at the early times and weak interaction later. Thus following the recipe provided in ref. [Padmanabhan, 2002] we can establish a model which can be a suitable alternative to dark energy explaining the decaying energy flow between the scalar field and the fluid and giving rise to a decaying potential. As a scalar field with potential to drive acceleration is a common practice in cosmology [Padmanabhan, 2002], the potential presented here can reproduce enough acceleration together with the ideal fluid, thus explaining the evolution of the

Universe. Also we have considered inhomogeneous EOS interacting with the scalar field which can represent an alternative to the usual dark energy model. However, stability analysis and spatial inhomogeneity analysis [Peebles etal, 1988; Rara etal, 1988] are more complicated for our investigation, since we are considering the ideal fluid with two types of equation of states and are analysing whether they can be considered as an alternative to dark energy. Also we have seen that the conservation equation (7.4) together with the given form of the pressure (7.6) and (7.12) are difficult to solve unless we consider the power law form (6.1). Once the power law form is considered, we can easily find exact solution of ρ_d [from equation (7.4))] and hence ρ_ϕ [from equation(4.2)], which lead to the given expression for the potential $V(\phi)$ [from eqs. (5.14), (5.15)] analytically. Though this is the backward approach, but otherwise if we start from $V(\phi)$ i.e., say $V(\phi) = V_0 Exp(-k\phi)$, we cannot find any exact solution of $\rho_d, \rho_\phi, p_d, p_\phi, \phi, a$. So we can only draw conclusions graphically, not analytically. For example, Ellis et al [1991] have discussed for the model with radiation and scalar field and found exact solutions in the backward approach.

Chapter 8

Perfect Fluid Dynamics in Brans-Dicke Theory

8.1 Prelude

Brans-Dicke (BD) theory has been proved to be very effective regarding the recent study of cosmic acceleration [Banerjee and Pavon, 2001]. As we have already discussed BD theory is explained by a scalar function ϕ and a constant coupling constant ω, often known as the BD parameter. This can be obtained from general theory of relativity (GR) by letting $\omega \to \infty$ and $\phi =$ constant [Sahoo and Singh, 2003]. This theory has very effectively solved the problems of inflation and the early and the late time behaviour of the Universe. Banerjee and Pavon [2001] have shown that BD scalar tensor theory can potentially solve the quintessence problem. The generalized BD theory [Bergmann, 1968; Nordtvedt, 1970; Wagoner, 1970] is an extension of the original BD theory with a time dependent coupling function ω. In Generalized BD theory, the BD parameter ω is a function of the scalar field ϕ. Banerjee and Pavon have shown that the generalized BD theory can give rise to a decelerating radiation model where the big-bang nucleosynthesis scenario is not adversely affected. Modified BD theory with a self-interacting potential have also been introduced in this regard. Bertolami and Martins [2000] have used this theory to present an accelerated Universe for spatially flat model. All these theories conclude that ω should have a low negative value in order to solve the cosmic acceleration problem. This contradicts the solar system experimental bound $\omega \geq 500$. However Bertolami and Martins [2000] have obtained the solution for accelerated expan-

sion with a potential ϕ^2 and large $|\omega|$, although they have not considered the positive energy conditions for the matter and scalar field.

In this chapter, we investigate the possibilities of obtaining accelerated expansion of the Universe in BD theory where we have considered a self-interacting potential V which is a function of the BD scalar field ϕ itself and a variable BD parameter which is also a function of ϕ. We show all the cases of $\omega =$ constant, $\omega = \omega(\phi)$, $V = 0$ and $V = V(\phi)$ to consider all the possible solutions. We examine these solutions for both barotropic fluid and the GCG, to get a generalized view of the results in the later case. We analyze the conditions under which we get a negative q (deceleration parameter, $-\frac{a\ddot{a}}{\dot{a}^2}$) in all the models of the Universe. For this purpose we have shown the graphical representations of these scenario for further discussion.

8.2 Field Equations

The self-interacting BD theory is described by the action (choosing $8\pi G_0 = c = 1$), given by equation (1.107), where $V(\phi)$ is the self-interacting potential for the BD scalar field ϕ and $\omega(\phi)$ is modified version of the BD parameter which is a function of ϕ [Sahoo and Singh, 2003]. The matter content of the Universe is composed of perfect fluid given by equation (1.42). From the action (1.107), we obtain the field equations (1.108), where,

$$\Box\,\phi = \frac{1}{3+2\omega(\phi)}T - \frac{1}{3+2\omega(\phi)}\left[2V(\phi) - \phi\frac{dV(\phi)}{d\phi}\right] - \frac{\frac{d\omega(\phi)}{d\phi}}{3+2\omega(\phi)}\phi_{,\mu}\phi^{,\mu} \tag{8.1}$$

and $T = T_{\mu\nu}g^{\mu\nu}$.

The line element for Friedman-Robertson-Walker space-time is given by equation (1.7). The Einstein field equations for the metric (1.7) and the wave equation for the BD scalar field ϕ are respectively given by equation (1.111), (1.112) and

$$\ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi}=\frac{\rho-3p}{3+2\omega(\phi)}+\frac{1}{3+2\omega(\phi)}\left[2V(\phi)-\phi\frac{dV(\phi)}{d\phi}\right]-\dot{\phi}\frac{\frac{d\omega(\phi)}{dt}}{3+2\omega(\phi)} \tag{8.2}$$

The energy conservation equation is (1.20).

Now we consider two types of fluids, first one being the barotropic perfect fluid and the second one is GCG.

8.3 Model with Barotropic Fluid in the Background

Here we consider the Universe to be filled with barotropic fluid with EOS

$$p=\gamma\rho \qquad\qquad (-1\leq\gamma\leq 1) \tag{8.3}$$

The conservation equation (1.20) yields the solution for ρ as,

$$\rho=\rho_0 a^{-3(\gamma+1)} \tag{8.4}$$

where $\rho_0(>0)$ is an integration constant.

8.3.1 Solution Without Potential

Case I: First we choose $\omega(\phi)=\omega=$ constant.

Now we consider power law form of the scale factor

$$a(t)=a_0 t^{\alpha} \qquad (\alpha\geq 1) \tag{8.5}$$

In view of equations (8.3) and (8.4), the wave equation (8.2) leads to the solution for ϕ to be

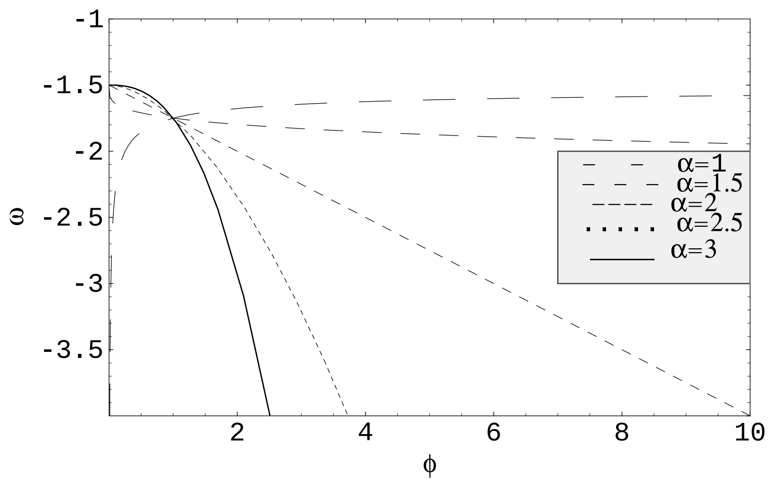

Fig 8.1: The variation of ω (for barotropic fluid) has been plotted against ϕ for different values of $\alpha = 1, 1.5, 2, 2.5, 3$ in a flat ($k = 0$) dust filled ($\gamma = 0$ and $\beta = -2(\text{d}ust)$) epoch, normalizing the parameters as $a_0 = \rho_0 = \phi_0 = 1$.

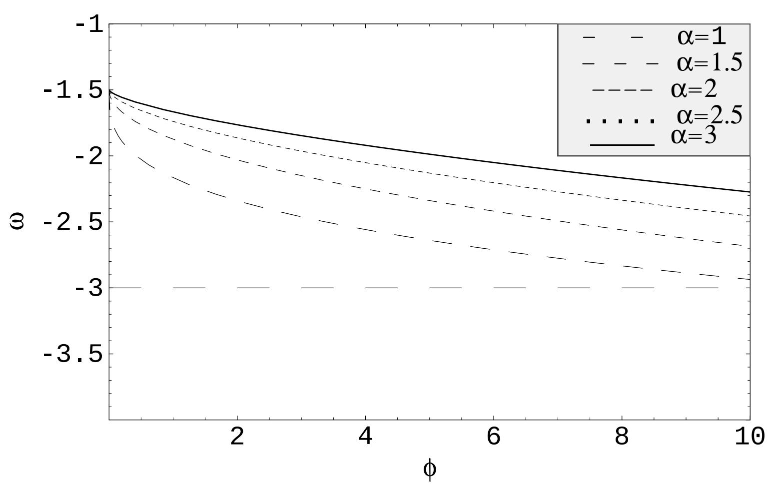

Fig 8.2: The variation of ω (for barotropic fluid) has been plotted against ϕ for different values of $\alpha = 1, 1.5, 2, 2.5, 3$ in a closed model ($k = 1$) in radiation ($\gamma = \frac{1}{3}$ and $\beta = -2\alpha$) era, normalizing the parameters as $a_0 = \phi_0 = 1,\ \rho_0 = 6$.

$$\phi = \frac{\rho_0 {a_0}^{-3(1+\gamma)} t^{2-3\alpha(1+\gamma)}}{(2\omega+3)(1-3\alpha\gamma)\left[2-3\alpha(1+\gamma)\right]} \tag{8.6}$$

For $k \neq 0$ we get from the field equations (1.111) and (1.112), the value of $\alpha = 1$ and

$$(3\gamma+1)\left[\frac{\omega}{2}(\gamma-1)(3\gamma+1) - 1 - \frac{k}{{a_0}^2}\right] = 0 \tag{8.7}$$

We have seen that $\gamma \neq -\frac{1}{3}$ and we have

$$\omega = \frac{2(1+\frac{k}{{a_0}^2})}{(\gamma-1)(3\gamma+1)} \tag{8.8}$$

Since ω must be negative for $-\frac{1}{3} < \gamma < 1$, we have seen that for this case the deceleration parameter $q = 0$, i.e., the universe is in a state of uniform expansion. For $k = 0$, the field equations yield

$$\left[2-3\alpha(\gamma+1)\right]\left[2(2\alpha-1) + \omega(\gamma-1)\{2-3\alpha(\gamma+1)\}\right] = 0 \tag{8.9}$$

From equation (8.9) we have two possible solutions for α:

$\alpha = \frac{2}{3(\gamma+1)}$ for $-1 < \gamma < -\frac{1}{3}$

and

$\alpha = \frac{2[1+\omega(1-\gamma)]}{[4+3\omega(1-\gamma^2)]}$ for $-\frac{1}{3} < \gamma < 1$

For these values of α, we have seen that $\omega < 0$ and the deceleration parameter $q < 0$. Thus for $k = 0$ with the power law form of the scale factor $a = a_0 t^\alpha$ it is possible to get the accelerated expansion of the Universe.

Case II: Now we choose $\omega = \omega(\phi)$ to be variable. Here we consider the power law form of ϕ as

$$\phi(t) = \phi_0 t^\beta \tag{8.10}$$

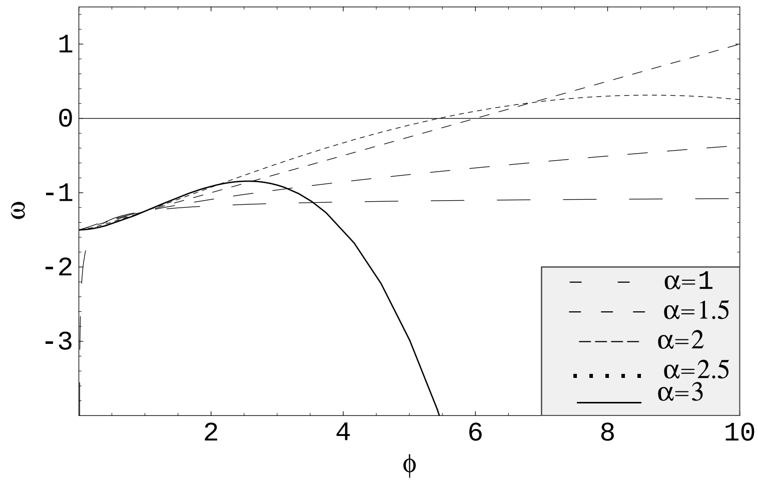

Fig 8.3: The variation of ω (for dust) has been plotted against ϕ for respectively closed model of the Universe in the present dust filled epoch, i.e., $\gamma = 0$ and $\beta = -2$. We take different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and normalize the parameters as $a_0 = \rho_0 = \phi_0 = 1$.

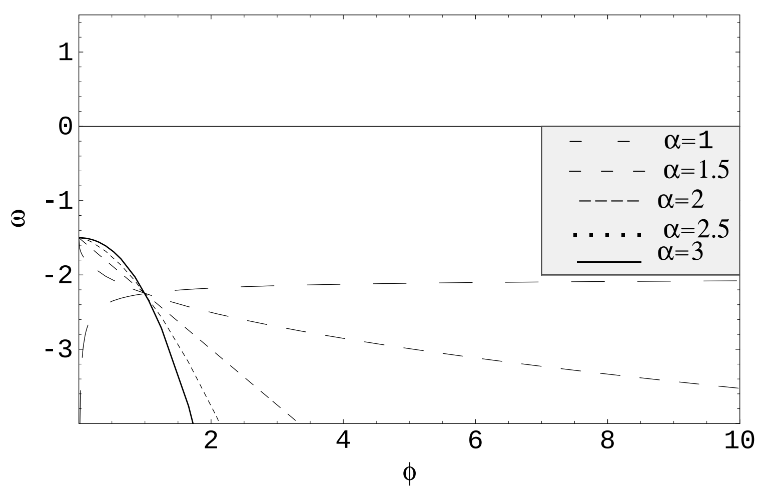

Fig 8.4: The variation of ω (for dust) has been plotted against ϕ for respectively open model of the Universe in the present dust filled epoch, i.e., $\gamma = 0$ and $\beta = -2$. We take different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and normalize the parameters as $a_0 = \rho_0 = \phi_0 = 1$.

with the power law from of $a(t)$ given by equation (8.5).

Proceeding as above we get

$$\omega = \frac{\alpha\beta + 2\alpha + \beta - \beta^2}{\beta^2} - \frac{1+\gamma}{\beta^2}\rho_0 {a_0}^{-3(1+\gamma)} {\phi_0}^{\frac{3\alpha(\gamma+1)-2}{\beta}} \phi^{-\frac{3\alpha(\gamma+1)+\beta-2}{\beta}} + \frac{2k}{{a_0}^2\beta^2{\phi_0}^{\frac{2(1-\alpha)}{\beta}}}\phi^{\frac{2(1-\alpha)}{\beta}} \tag{8.11}$$

Now for acceleration $q < 0$ implies that $\alpha > 1$. Using the other equations we arrive at two different situations:

(i) First considering the flat Universe model, i.e., $k = 0$, we get, $\beta = 1 - 3\alpha$, i.e., $\beta < -2$ for $\gamma > \frac{1}{3}$, $\beta = -2\alpha$, i.e., $\beta < -2$ (as $\alpha > 1$) for $\gamma = \frac{1}{3}$ and $\beta = -2$ for $\gamma < \frac{1}{3}$. That is cosmic acceleration can be explained at all the phases of the Universe with different values of β where $\phi = \phi_0 t^\beta$

(ii) If we consider the non-flat model of the Universe, i.e., $k \neq 0$, we are left with two options. For closed model of the Universe, i.e., for $k = 1$ we can explain cosmic acceleration for the radiation phase only and for that $\beta = -2\alpha$ giving $\beta < -2$ and $6\phi_0 {a_0}^2 = \rho_0$, whereas we do not get any such possibility for the open model of the Universe.

Now preferably taking into account the recent measurements confirming the flat model of the Universe, if $\beta = -2$ we see that we have an accelerated expansion of the Universe after the radiation period preceded by a decelerated expansion before the radiation era and a phase of uniform expansion at the radiation era itself. Also if $\beta < -2$ cosmic acceleration is followed by a deceleration phase as $\alpha < 1$ for $\gamma < \frac{1}{3}$.

8.3.2 Solution With Potential

Case I: Let us choose $\omega(\phi) = \omega =$ constant.

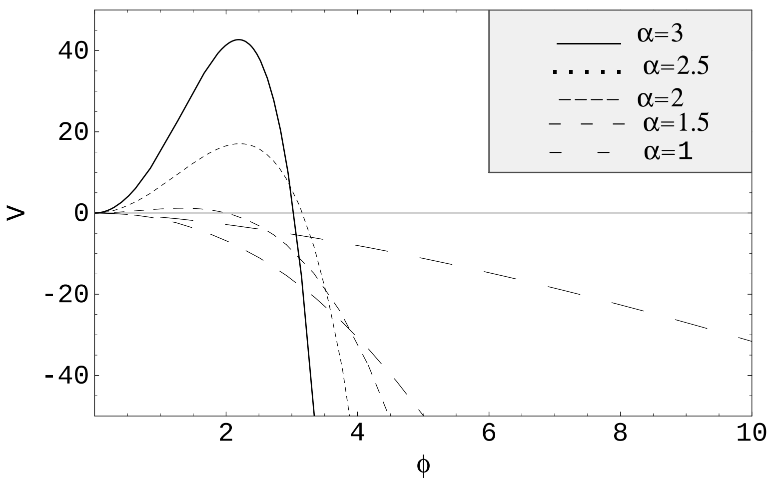

Fig 8.5: Variation of V (for dust) has been plotted against ϕ for flat ($k=0$) model of the Universe in dust filled epoch ($\gamma=0$). We have considered different values of $\alpha=1,1.5,2,2.5,3$ and $\beta=-2$ and normalize the parameters as $a_0=\rho_0=\phi_0=1$.

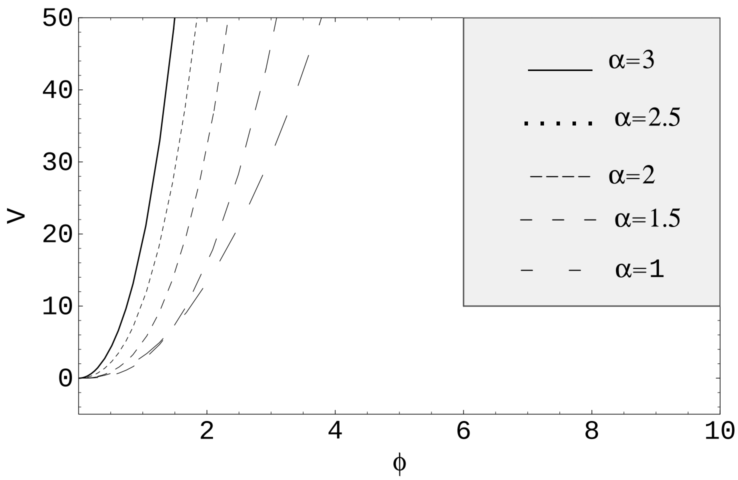

Fig 8.6: Variation of V (for dust) has been plotted against ϕ for closed ($k=1$) model of the Universe in dust filled epoch ($\gamma=0$). We have considered different values of $\alpha=1,1.5,2,2.5,3$ and $\beta=-2$ and normalize the parameters as $a_0=\rho_0=\phi_0=1$.

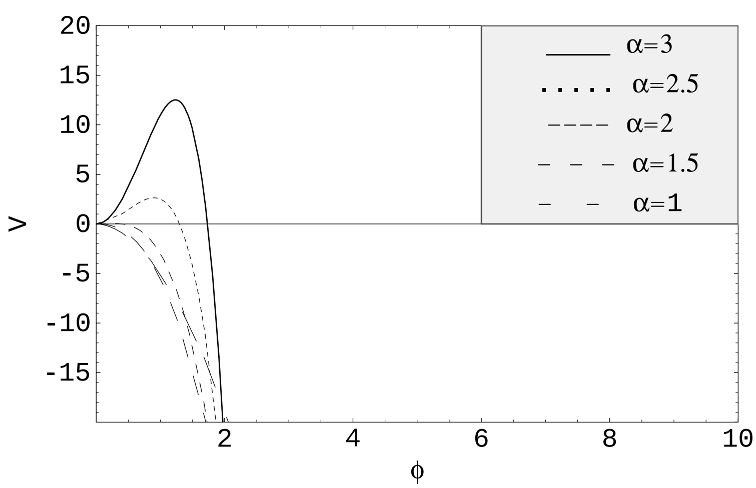

Fig 8.7: Variation of V (for dust) has been plotted against ϕ for open ($k=-1$) model of the Universe in dust filled epoch ($\gamma=0$). We have considered different values of $\alpha=1,1.5,2,2.5,3$ and $\beta=-2$ and normalize the parameters as $a_0=\rho_0=\phi_0=1$.

In this case instead of considering equations (8.5) and (8.10) we consider only one power law form

$$\phi = \phi_0 a^\alpha \tag{8.12}$$

Using equation (8.12) in equations (1.111) and (1.112) we get

$$\dot{a} = \left[2k + 2(1+\gamma)\frac{\rho_0}{\phi_0}\frac{a^{-3\gamma-\alpha-1}}{\{3\gamma\alpha + 6\gamma - \alpha^2 + 7\alpha + 6 - 2\omega\alpha^2\}}\right]^{\frac{1}{2}}$$

Putting $k = 0$, we get

$$a = At^{\frac{2}{3+\alpha+3\gamma}} \tag{8.13}$$

where $A = \left[\frac{\rho_0(1+\gamma)(3+\alpha+3\gamma)^2}{2\rho_0\{6(1+\gamma)+\alpha(7+3\gamma)-\alpha^2(1+2\omega)\}}\right]^{\frac{1}{3+\alpha+3\gamma}}$.

Therefore, $\phi = Bt^{\frac{2\alpha}{3+\alpha+3\gamma}}$ where, $B = \phi_0 A^\alpha$.

Now, if $\frac{2}{3+\alpha+3\gamma} \geq 1$, we get

$$\alpha \leq -(1+3\gamma) \tag{8.14}$$

Substituting these values in (1.111), (1.112), (8.2), the solution for the potential V is obtained as, $V = \frac{B'}{\phi^{\frac{3+3\gamma}{\alpha}}}$ where, $B' = -\frac{2B\{6-18\alpha+6\omega\alpha+6\omega\alpha\gamma-18\gamma}{3(3+3\gamma+\alpha)^2(1+\gamma)}$.

Also, the deceleration parameter reduces to, $q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{3\gamma+\alpha+1}{2} \leq 0$ (using equation (8.14))

Hence, the present Universe is in a state of expansion with acceleration.

Also, we get $\omega=-\frac{6\gamma(1+\gamma)}{\alpha}-\frac{3+\alpha}{2\alpha}$ and, $\alpha=-\frac{3(1+2\gamma)^2}{1+2\omega}$.

Also, $\gamma\geq-1\Rightarrow\alpha\leq 2$ and $\omega\geq-\frac{5}{4}$. For the present Universe (i.e., taking $\gamma=0$) and the ΛCDM model, $\omega=-\frac{3+\alpha}{2\alpha}$.

Case II: Now we choose $\omega(\phi)$ to be dependent on ϕ. Again we consider the power law forms (8.5) and (8.10). Solving the equations in a similar manner, we get

$$\omega=\frac{\alpha\beta+2\alpha+\beta-\beta^2}{\beta^2}-\frac{1+\gamma}{\beta^2}\rho_0{a_0}^{-3(1+\gamma)}{\phi_0}^{\frac{3\alpha(\gamma+1)-2}{\beta}}\phi^{-\frac{3\alpha(\gamma+1)+\beta-2}{\beta}}+\frac{2k}{{a_0}^2\beta^2{\phi_0}^{\frac{2(1-\alpha)}{\beta}}}\phi^{\frac{2(1-\alpha)}{\beta}} \tag{8.15}$$

and

$$V(\phi)=(2\alpha+\beta)(3\alpha+2\beta-1){\phi_0}^{\frac{2}{\beta}}\phi^{\frac{\beta-2}{\beta}}-(1-\gamma)\rho_0{a_0}^{-3(1+\gamma)}{\phi_0}^{\frac{3\alpha(\gamma+1)}{\beta}}\phi^{-\frac{3\alpha(\gamma+1)}{\beta}}+\frac{4k}{{a_0}^2}\phi^{\frac{\beta-2\alpha}{\beta}}{\phi_0}^{\frac{2\alpha}{\beta}} \tag{8.16}$$

Substituting these values in equation (8.2), we get

$$\text{either}\quad \beta=-2\quad\text{or}\quad\beta=-2\alpha \tag{8.17}$$

Therefore for cosmic acceleration $q<0\Rightarrow\alpha>1$ and $\beta\leq-2$.

Therefore for the present era,

$$\omega=-\frac{3}{2}-\frac{\rho_0{a_0}^{-3}}{4\phi_0}t^{\frac{3\alpha-4}{2}}\quad\text{and}\quad V=2(\alpha-1)(3\alpha-5)\phi_0 t-t^{\frac{3\alpha}{2}}\rho_0{a_0}^{-3}\quad\text{if}\quad\beta=-2$$

$$\omega=-\frac{3}{2}-\frac{\rho_0{a_0}^{-3}}{\phi_0}t^{\frac{\alpha-2}{2\alpha}}\quad\text{and}\quad V=-\phi_0{a_0}^{-3}t^{\frac{3}{2}}\quad\text{if}\quad\beta<-2 \tag{8.18}$$

Also for vacuum dominated era,

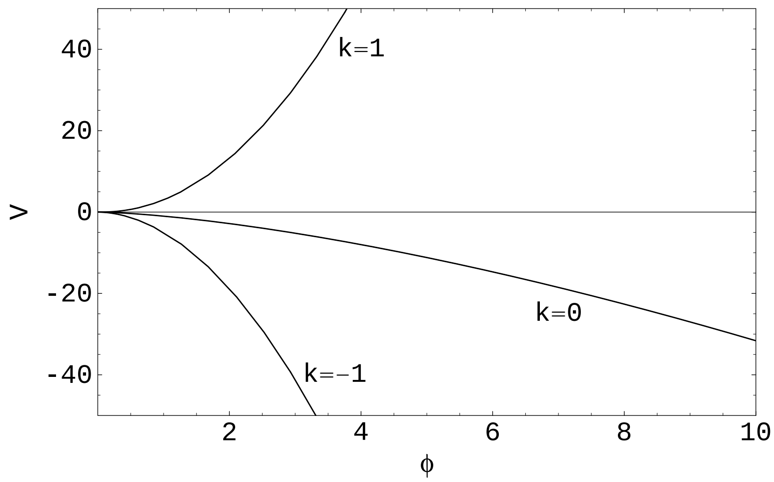

Fig 8.8: Variation of V (for dust) is plotted against the variation of ϕ for all the models of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and the present dust filled epoch, i.e., $\gamma = 0$, normalizing the parameters as $a_0 = \rho_0 = \phi_0 = 1$ and $\beta = -2\alpha$. The results for different values of α coincides with each other in each model of the Universe.

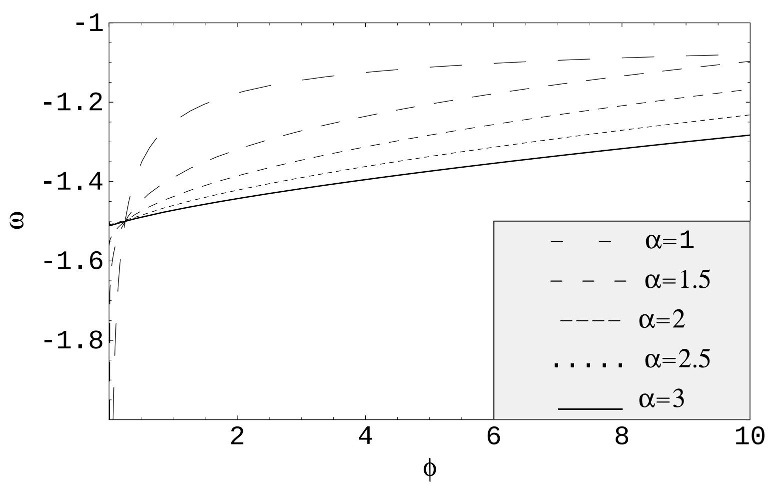

Fig 8.9: Variation of ω (for dust) is plotted for closed ($k = 1$) model of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and the present dust filled epoch, i.e., $\gamma = 0$, normalizing the parameters as $a_0 = \rho_0 = \phi_0 = 1$ and $\beta = -2\alpha$.

$$\omega = -\frac{3}{2} \quad \text{and} \quad V = 2(\alpha-1)(3\alpha-5)t - 2\rho_0 \quad \text{for} \quad \beta = -2$$

$$\omega = -\frac{3}{2} \quad \text{and} \quad V = -2\rho_0 \quad \text{for} \quad \beta < -2 \tag{8.19}$$

8.4 Model with GCG in the Background

Here we consider the Universe to be filed with Generalized Chaplygin Gas with EOS

$$p = -\frac{B}{\rho^n} \tag{8.20}$$

Here the conservation equation (1.20) yields the solution for ρ as,

$$\rho = \left[B + \frac{C}{a^{3(1+n)}}\right]^{\frac{1}{(1+n)}} \tag{8.21}$$

where C is an integration constant.

8.4.1 Solution Without Potential

Case I: First we choose $\omega(\phi) = \omega =$ constant.

We consider the power law form

$$\phi = \phi_0 a^{\alpha} \tag{8.22}$$

Equations (1.111), (1.112) and (8.2) give,

$$(2\omega\alpha - 6)\ddot{a} + (\omega\alpha^2 + 4\omega\alpha - 6)\frac{\dot{a}^2}{a} = \frac{6k}{a} \tag{8.23}$$

which yields the solution,

$$\dot{a} = \sqrt{\frac{6k}{P(\omega\alpha - 3)} + K_0 a^{-P}} \tag{8.24}$$

where $P=\frac{\omega\alpha^2+4\omega\alpha-6}{\omega\alpha-3}$ and K_0 is an integration constant.

First we consider $P>0$. Multiplying both sides of equation (8.24) by a^P after squaring it, we get $K_0=0$, therefore giving, $a=\sqrt{\frac{6k}{(\omega\alpha-3)P}}t$.

Hence for flat Universe, we get, $a=$ constant.

For open model, we must have $\omega\alpha<3$ and $a=\sqrt{\frac{6}{(3-\omega\alpha)P}}t$, whereas, for closed model, $\omega\alpha>3$ and $a=\sqrt{\frac{6}{(\omega\alpha-3)P}}t$. In all cases $q=0$, i.e., we get uniform expansion.

If $P=0$, $a\ddot{a}=\frac{3}{\omega\alpha-3}k$, i.e., $\dot{a}^2=\frac{6k}{\omega\alpha-3}\ln a+K_0$.

If $k=0$, $a=\sqrt{K_0}t+C_0$, (C_0 is an integration constant) causing $q=0$, i.e., uniform expansion again.

Case II: Now we consider $\omega=\omega(\phi)$, i.e., ω dependent on ϕ.

Also the power law forms considered will be (8.5) and (8.10). Solving the equations we get,

$$\omega(\phi)=\frac{\alpha\beta+2\alpha+\beta-\beta^2}{\beta^2}-\frac{C{a_0}^{-3(1+n)}{\phi_0}^{\frac{3\alpha(1+n)-2}{\beta}}\phi^{\frac{-3\alpha(1+n)-\beta+2}{\beta}}}{\beta^2\left[B+C{a_0}^{-3(1+n)}{\phi_0}^{\frac{3\alpha(1+n)}{\beta}}\phi^{\frac{-3\alpha(1+n)}{\beta}}\right]^{\frac{n}{1+n}}}+\frac{2k\phi^{\frac{2(1-\alpha)}{\beta}}}{{a_0}^2\beta^2{\phi_0}^{\frac{2(1-\alpha)}{\beta}}} \tag{8.25}$$

Also substituting these values in the given equations, we get, either $n=-1$ or $B=0$ and also $k=0$. If $n=-1$, we get back barotropic fluid, and if $B=0$, we get dust filled Universe. In both the cases the Generalized Chaplygin gas does not seem to have any additional effect on the cosmic acceleration.

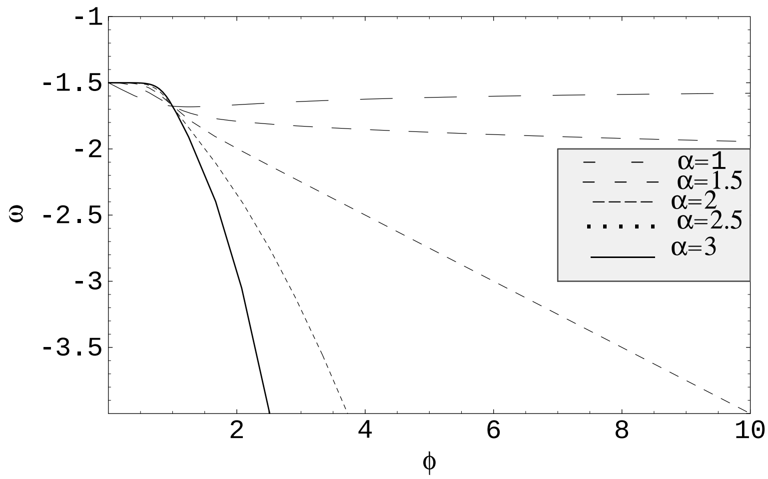

Fig 8.10: Variation of ω (for GCG) is shown against ϕ for flat ($k=0$) model of the Universe. We have considered different values of $\alpha=1,1.5,2,2.5,3$ and $\beta=-2$ and normalize the parameters as $a_0=\rho_0=\phi_0=B=C=1$ and $n=1$.

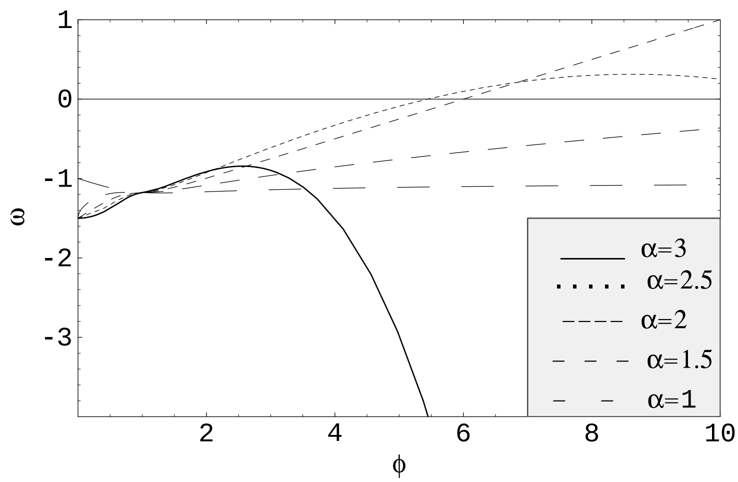

Fig 8.11: Variation of ω (for GCG) is shown against ϕ for closed ($k=1$) model of the Universe. We have considered different values of $\alpha=1,1.5,2,2.5,3$ and $\beta=-2$ and normalize the parameters as $a_0=\rho_0=\phi_0=B=C=1$ and $n=1$.

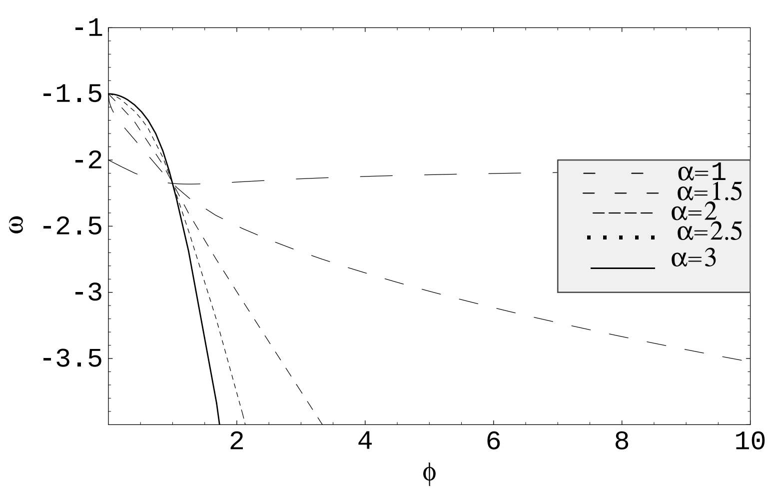

Fig 8.12: Variation of ω (for GCG) is shown against ϕ for open ($k=-1$) model of the Universe. We have considered different values of $\alpha=1,1.5,2,2.5,3$ and $\beta=-2$ and normalize the parameters as $a_0=\rho_0=\phi_0=B=C=1$ and $n=1$.

8.4.2 Solution With Potential

Case I: Let us choose $\omega(\phi) = \omega =$ constant.

We again consider the power law forms (8.5) and (8.10). We get the solution for $V(\phi)$ to be

$$V(\phi) = (2\alpha+\beta)(3\alpha+2\beta-1){\phi_0}^{\frac{2}{\beta}}\phi^{\frac{\beta-2}{\beta}}+\frac{-2B-C{a_0}^{-3(1+n)}{\phi_0}^{\frac{3\alpha(1+n)}{\beta}}\phi^{\frac{-3\alpha(1+n)}{\beta}}}{\left[B+C{a_0}^{-3(1+n)}{\phi_0}^{\frac{3\alpha(1+n)}{\beta}}\phi^{\frac{-3\alpha(1+n)}{\beta}}\right]^{\frac{n}{(1+n)}}}+\frac{4k}{{a_0}^2}\phi^{\frac{\beta-2\alpha}{\beta}}{\phi_0}^{\frac{2\alpha}{\beta}} \tag{8.26}$$

Substituting these values in the other equations we get that $n = -1$, i.e., the equation of state of Generalized Chaplygin Gas takes the form of that of barotropic fluid. Also we get, $\alpha = 1$, which implies $q = 0$, i.e., uniform expansion of the Universe.

Case II: Now we choose $\omega(\phi)$ to be dependent on ϕ.

Again we consider the power law forms, (8.5) and (8.10). Solving the equations we get the solutions for Brans-Dicke parameter and self-interacting potential as same as equations (8.25) and (8.26) respectively.

Substituting these values in equation (8.2), we get

$$\text{either} \quad \beta = -2 \quad \text{or} \quad \beta = -2\alpha \tag{8.27}$$

Therefore for cosmic acceleration $q < 0 \Rightarrow \alpha > 1$ and $\beta \leq -2$.

Therefore for the dust dominated era,

$$\omega = -\frac{3}{2} - \frac{\rho_0 {a_0}^{-3}\phi^{\frac{3\alpha-4}{2}}}{4{\phi_0}^{\frac{3\alpha-2}{2}}} \qquad \text{and} \quad V = \frac{2(\alpha-1)(3\alpha-5)}{\phi_0}\phi^2 - \rho_0 {a_0}^{-3}\frac{\phi^{\frac{3\alpha}{2}}}{{\phi_0}^{\frac{3\alpha}{2}}} \quad \text{if} \quad \beta = -2$$

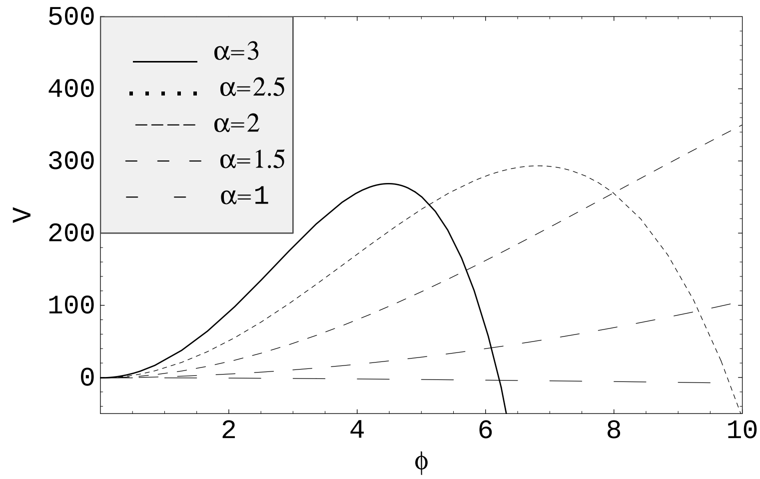

Fig 8.13: Variation of V (for GCG) has been plotted against ϕ for flat ($k = 0$) model of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and $\beta = -2$ and normalize the parameters as $a_0 = \rho_0 = \phi_0 = B = C = 1$ and $n = 1$.

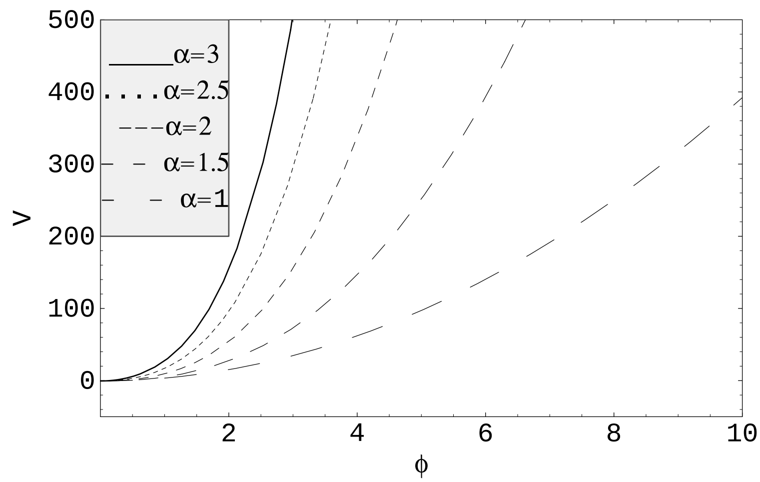

Fig 8.14: Variation of V (for GCG) has been plotted against ϕ for closed ($k = 1$) model of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and $\beta = -2$ and normalize the parameters as $a_0 = \rho_0 = \phi_0 = B = C = 1$ and $n = 1$.

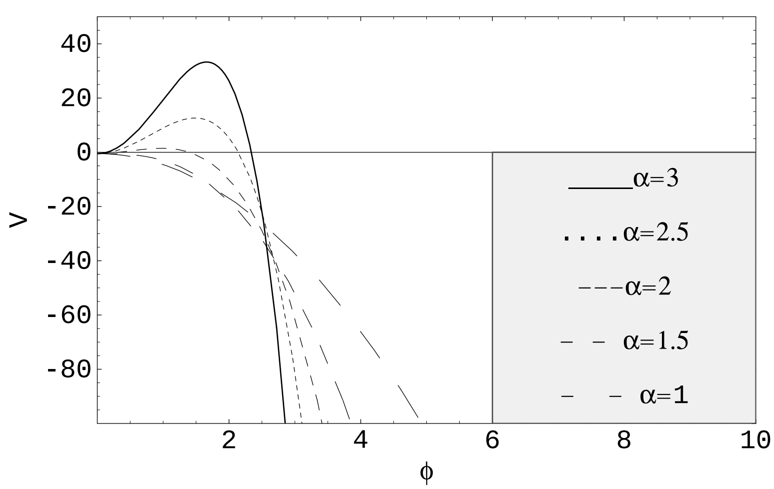

Fig 8.15: Variation of V (for GCG) has been plotted against ϕ for open ($k = -1$) model of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$ and $\beta = -2$ and normalize the parameters as $a_0 = \rho_0 = \phi_0 = B = C = 1$ and $n = 1$.

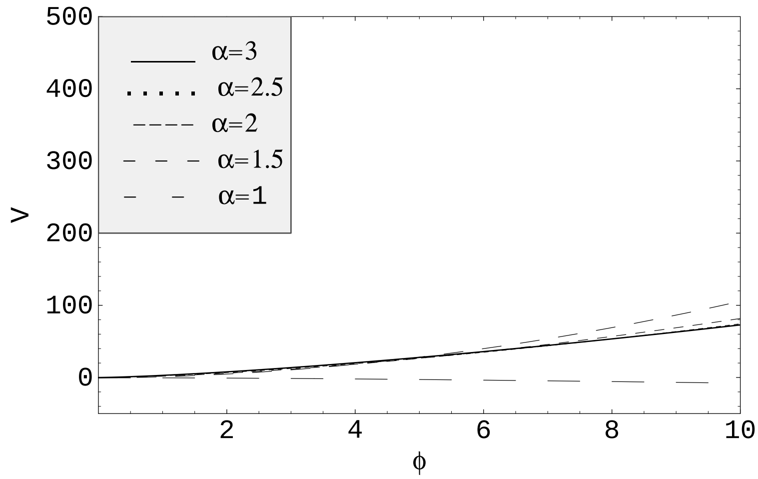

Fig 8.16: The variation of V (for GCG) is shown against ϕ for flat ($k=0$) model of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$, $n = 1$ and $\beta = -2\alpha$ and normalize the parameters as $a_0 = \rho_0 = \phi_0 = B = C = 1$.

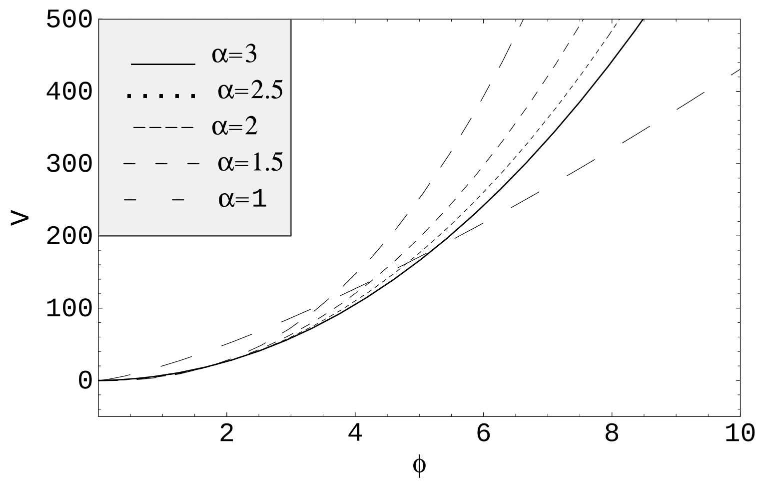

Fig 8.17: The variation of V (for GCG) is shown against ϕ for closed ($k=1$) model of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$, $n = 1$ and $\beta = -2\alpha$ and normalize the parameters as $a_0 = \rho_0 = \phi_0 = B = C = 1$.

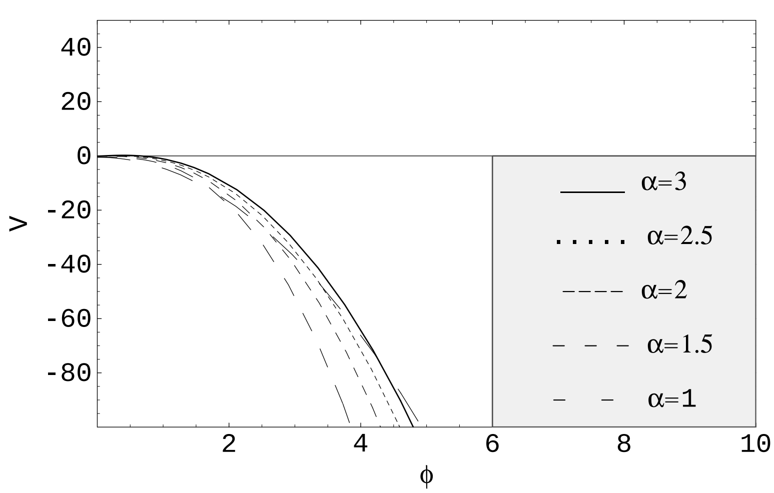

Fig 8.18: The variation of V (for GCG) is shown against ϕ for open ($k=-1$) model of the Universe. We have considered different values of $\alpha = 1, 1.5, 2, 2.5, 3$, $n = 1$ and $\beta = -2\alpha$ and normalize the parameters as $a_0 = \rho_0 = \phi_0 = B = C = 1$.

$$\omega = -\frac{3}{2} - \frac{\rho_0 {a_0}^{-3} \phi^{\frac{\alpha-2}{2\alpha}}}{4\alpha^2 {\phi_0}^{\frac{3\alpha-2}{2\alpha}}} \quad \text{and} \quad V = -\rho_0 {a_0}^{-3} \frac{\phi^{\frac{3}{2}}}{{\phi_0}^{\frac{3}{2}}} \quad \text{if} \quad \beta < -2 \tag{8.28}$$

Also for vacuum dominated era,

$$\omega = -\frac{3}{2} \quad \text{and} \quad V = 2(\alpha-1)(3\alpha-5)\frac{\phi^2}{\phi_0} - 2\left[\rho_{vac}\right]_{\beta=-2} \quad \text{for} \quad \beta = -2$$

$$\omega = -\frac{3}{2} \quad \text{and} \quad V = -2\left[\rho_{vac}\right]_{2\alpha+\beta=0} \quad \text{for} \quad \beta < -2 \tag{8.29}$$

8.5 Discussion

We are considering Friedman-Robertson-Walker model in Brans-Dicke Theory with and without potential (V). Also we have considered the Brans-Dicke parameter (ω) to be constant and variable. We take barotropic fluid and Generalized Chaplygin Gas as the concerned fluid.

Using barotropic equation of state, we get,

(i) for $V = 0$ and $\omega =$constant, $\omega < 0$ and $q < 0$ for some values of α, giving rise to cosmic acceleration,

(ii) for $V = 0$ and $\omega = \omega(\phi)$, we obtain cosmic acceleration depending on some values of α and β. In this case we get acceleration for closed model also at the radiation phase. We can show the variation of $\omega(\phi)$ against the variation of ϕ here [figure 8.1, 8.2]. Figure 8.1 shows that as the value of α increases ω decreases steadily against the variation of ϕ. For $\alpha > 1$, we have accelerated expansion. The figure shows that the greatest value of ω can be $-\frac{3}{2}$ and it decreases further as ϕ increases,

(iii) for $V = V(\phi)$ and $\omega =$constant, we get acceleration in the flat model irrespective of the values of α,

(iv) for $V = V(\phi)$ and $\omega = \omega(\phi)$, cosmic acceleration is obtained for $\beta \leq 2$. Here we can represent the variation of ω and V against the variation of ϕ for $\beta = -2$ and $\beta = -2\alpha$. For $\beta = -2$, the variation of ω against ϕ is same as figure 8.1 and that for closed and

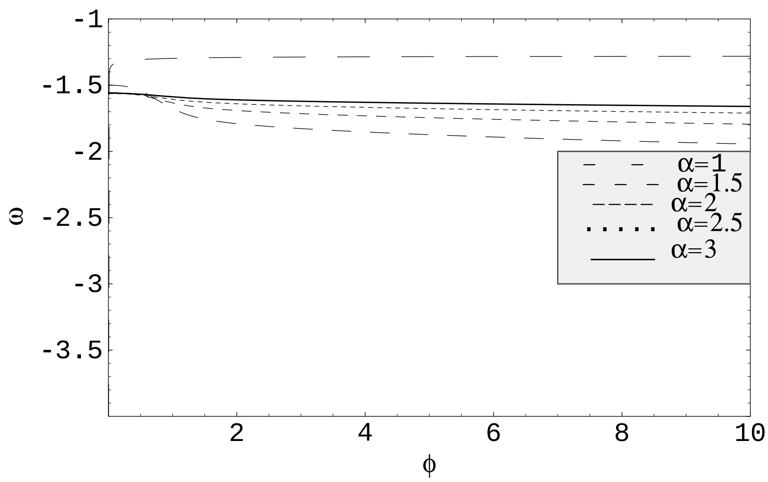

Fig 8.19: Variation of ω (for GCG) is shown against ϕ for different values of $\alpha = 1, 1.5, 2, 2.5, 3$ in a flat ($k = 0$) model of the Universe. Here we have considered $\beta = -2\alpha$ and $n = 0$ and normalize the parameters as $a_0 = \rho_0 = \phi_0 = b = C = 1$.

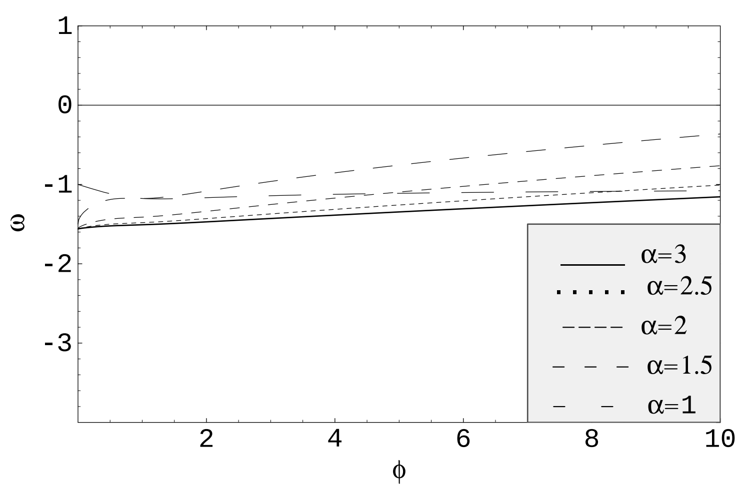

Fig 8.20: Variation of ω (for GCG) is shown against ϕ for different values of $\alpha = 1, 1.5, 2, 2.5, 3$ in a closed ($k = 1$) model of the Universe. Here we have considered $\beta = -2\alpha$ and $n = 0$ and normalize the parameters as $a_0 = \rho_0 = \phi_0 = b = C = 1$.

open models are given in figure 8.3 and 8.4. Here we can see that for open model ω starting at $-\frac{3}{2}$ decreases further, whereas for closed model ω starting at $-\frac{3}{2}$ increases to be positive for $\alpha = 2$. Figures 8.5, 8.6 and 8.7 show that variation of V against the variation of ϕ for $\beta = -2$ in respectively flat, closed and open models of the Universe. Here we can see that only for the closed model the potential increases positively, in the other two cases the potential becomes negative after a certain point. Figure 8.8 shows the variation of V against ϕ for $\beta = -2\alpha$. Again positive potential energy is obtained for only the closed model. The variation of ω is shown in figure 8.9 for $k = 1$ and we can see that ω increases starting at $-\frac{3}{2}$.

Using Generalized Chaplygin Gas , we get,

(i) for $V = 0$ and $\omega =$constant, uniform expansion is obtained,

(ii) for $V = 0$ and $\omega = \omega(\phi)$, Generalized Chaplygin Gas does not seem to have any effect of itself,

(iii) for $V = V(\phi)$ and $\omega =$constant, we get $q = 0$ giving uniform expansion,

(iv) for $V = V(\phi)$ and $\omega = \omega(\phi)$ cosmic acceleration is obtained for $\beta \leq 2$ as previously obtained for barotropic fluid. Figures 8.10, 8.11, and 8.12 show the variation of ω for $\beta = -2$ in flat closed and open models and the natures of the graphs do not vary much from that for barotropic fluid. Figures 8.13, 8.14 and 8.15 show the variation of V for flat, closed and open models respectively. Here for open model we get a negative potential after a certain point, whereas for closed model we get a positive potential always. For spatially flat model a positive V is obtained for $\alpha = 1.5, 2$. Figures 8.16, 8.17 and 8.18 show the variation of V for the models of the Universe for $\beta = -2\alpha$. Positive potential is obtained for closed model and flat model shows positive potential for $\alpha > 1$. For open model we get negative V again. Figures 8.19 and 8.20 show the variation of ω for flat and closed models respectively ($\beta = -2\alpha$). For flat model ω starting at $-\frac{3}{2}$ decreases further and for closed model it increases slowly from $-\frac{3}{2}$.

We have used BD theory to solve the problem of cosmic acceleration. Here we use barotropic fluid and Generalized Chaplygin Gas. Although the problem of fitting the value of ω to the limits imposed by the solar system experiments could not be solved fully, for closed Universe and $\beta = -2$ and $\alpha > 1$, ω starting from $-\frac{3}{2}$ increases and for large ϕ, we get $\omega > 500$, for both barotropic fluid and Generalized Chaplygin Gas. Also for flat Universe filled with barotropic fluid taking $\omega =$constant and $V = V(\phi)$, we get the Bertolami-Martins [2000] solution, i.e, $V = V(\phi^2)$ and $q_0 = -\frac{1}{4}$ for $a = At^{\frac{4}{3}}$. But taking Generalized Chaplygin Gas, we get accelerated expansion only when both ω and V are functions of the scalar field ϕ. For $\beta = -2$ we get cosmic acceleration in the closed model, whereas, $\beta = -2\alpha$ gives acceleration in both closed and flat models of the Universe, although for flat Universe ω varies from $-\frac{3}{2}$ to -2 and for closed Universe ω takes large values for large ϕ. In the end we see that for all the cases accelerated expansion can be achieved for closed model of the Universe for large values of ω. Also the present day acceleration of the Universe can also be explained successfully, although in this case ω cannot meet the solar system limits.

Short Discussions and Concluding Remarks

This thesis concentrates on the accelerated expansion of the Universe recently explored by measurements of redshift and luminosity-distance relations of type Ia Supernovae. This work also deals with the dark energy problem, which is recently one of the most widely investigated problems in Cosmology. A few dark energy models have been considered for this purpose. These models have been discussed, the equations have been solved for exact solutions to show the significance of these models to solve the dark energy problem. Statefinder diagnostics play very important role here, as the statefinder parameters have been solved and plotted to show the evolution of the Universe.

In chapter 1, standard cosmology and the FRW model of the Universe have been described. It also discusses dark matter and dark energy and various candidates of dark energy. A brief introduction to Brans-Dicke cosmology has also been given. This chapter ends with a short note on the statefinder parameters.

Chapter 2 deals with a model of the universe filled with modified Chaplygin gas and barotropic fluid. The field equations have been solved to show its role in acceleration of the universe. Statefinder parameters have been solved and plotted to show the different phases of evolution of the Universe. This model has also been discussed from field theoretical point of view.

In chapter 3 the role of dynamical cosmological constant has been explored with Modified Chaplygin Gas as the background fluid. Various phenomenological models for Λ, viz., $\Lambda \propto \rho, \Lambda \propto \frac{\dot{a}^2}{a^2}$ and $\Lambda \propto \frac{\ddot{a}}{a}$ have been considered for this purpose. These models have been studied in presence of the gravitational constant G to be constant or time dependent. Natures of G and Λ have been shown over the total age of the Universe. Statefinder analysis has been done to show the evolution of the Universe.

Recently developed Generalized Cosmic Chaplygin gas (GCCG) is studied in chapter 4 as an unified model of dark matter and dark energy. To explain the recent accelerating phase, the Universe is assumed to have a mixture of radiation and GCCG. The mixture is considered for without or with interaction. Solutions are obtained for various choices of the parameters and trajectories in the plane of the statefinder parameters and presented graphically. For particular choice of interaction parameter, the role of statefinder parameters have been shown in various cases for the evolution of the Universe.

In chapter 5 a new form of the well known Chaplygin gas model has been presented by introducing inhomogeneity in the EOS. This model explains $\omega = -1$ crossing. Also a graphical representation of the model using $\{r, s\}$ parameters have been given to show the evolution of the Universe. An interaction of this model with the scalar field has also been investigated through a phenomenological coupling function. A decaying nature of the potential has been shown for this model.

In chapter 6 tachyonic field has been depicted as dark energy model to represent the present acceleration of the Universe. For this purpose a mixture of tachyonic fluid with barotropic fluid has been assumed. Also a mixture of the tachyonic fluid has been considered with Generalized Chaplygin Gas to show the role of the later as a dark energy candidate in presence of tachyonic matter. A particular form of the scale factor has been assumed to solve the equations of motion and get the exact solutions of the density, tachyonic potential and the tachyonic field. A coupling term has also been introduced in both the models to represent the energy transfer between the two fluids. The interaction term and the tachyonic potential has been analysed and plotted to show their nature in the evolution of the Universe.

Chapter 7 deals with inhomogeneous EOS. A model of interaction has been studied with scalar field and the inhomogeneous ideal fluid. Two forms of the ideal fluid have

been analysed. A power law expansion for the scale factor has been assumed to solve the equations for the energy densities. This model shows a decaying nature of the scalar field potential and the interaction parameter.

In chapter 8, Brans-Dicke theory has been used to investigate the possibility of obtaining cosmic acceleration. For this purpose a constant and a variable ω (Brans-Dicke parameter) have been considered. A self-interacting potential has been introduced to show its role in the evolution of the Universe. This model has been studied in presence of barotropic fluid and Generalized Chaplygin Gas. Power law forms of the scale factor and the scalar field have been assumed to solve the field equations. It has been shown that accelerated expansion can also be achieved for high values of ω for closed Universe.

References of the Papers Presented in the Thesis

1. **IS MODIFIED CHAPLYGIN GAS ALONG WITH BAROTROPIC FLUID RESPONSIBLE FOR ACCELERATION OF THE UNIVERSE?: -**

   Writambhara Chakraborty and Ujjal Debnath

   Modern Physics Letters A, **22** 1805-1812 (2007).

   Pre-print: *gr-qc*/ 0611094.

2. **GENERALIZED COSMIC CHAPLYGIN GAS MODEL WITH R WITHOUT INTERACTION: -**

   Writambhara Chakraborty, Ujjal Debnath and Subenoy Chakraborty

   Gravitation and Cosmology, **13** 293-297 (2007).

   Pre-print: *gr-qc*/ 0711.0079.

3. **Effect of dynamical cosmological constant in presence of modified Chaplygin gas for accelerating universe: -**

   Writambhara Chakraborty and Ujjal Debnath

   Astrophysics and Space Science, **313** 409-417 (2008).

   Pre-print: *gr-qc*/ 0705.4147.

4. **Interaction between scalar field and ideal fluid with inhomogeneous equation of state: -**

   Writambhara Chakraborty and Ujjal Debnath

   Physics Letters B, **661** 1-4 (2008).

   Pre-print: *gr-qc*/ 0802.3751.

5. **Role of a tachyonic field in accelerating the Universe in the presence of a perfect fluid: -**

   Writambhara Chakraborty and Ujjal Debnath

   Astrophysics and Space Science, **315** 73-78 (2008).

   Pre-print: *gr-qc*/ 0804.4801.

6. **Role of Brans-Dicke Theory with or without Self-Interacting Potential in Cosmic Acceleration: -**

   Writambhara Chakraborty and Ujjal Debnath

   International Journal of Theoretical Physics, **48** 232-247 (2009).

   Pre-print: *gr-qc*/ 0807.1776.

7. **A New Variable Modified Chaplygin Gas Model Interacting with Scalar Field: -**

   Writambhara Chakraborty and Ujjal Debnath

   (Communicated in *Gravitation and Cosmology*)

Bibliography

[1] Abdel-Rahman, A. M. M. (1990) *Gen. Rel. Grav.* **22**, 655.

[2] Abdel-Rahman, A. M. M. (1992) *Phys. Rev. D* **45**, 3497.

[3] Abdussattar and Vishwakarma, R. G. (1997) *Class. Quantum Grav.* **14**, 945.

[4] Adler S. L. (1980) *Phys. Rev. Lett.* **44**, 1567.

[5] Alam, U., Sahni, V., Saini, T. D. and Starobinsky, A. A. (2003) *Mon. Not. Roy. Astron. Soc.* **344**, 1057.

[6] Albrecht, A. and Skordis, C. (2000) *Phys. Rev. Lett.* **84**, 2076.

[7] Al-Rawaf, A. S. and Taha, M. O. (1996) *Gen. Rel. Grav.* **28**, 935.

[8] Amendola, L. (2000) *Phys. Rev. D* **62**, 043511.

[9] Arbab I. Arbab (1997) *Gen. Rel. Grav.* **29**, 61.

[10] Arbab I. Arbab (1998) *Astrophys. Space Sci.* **259**, 371.

[11] Arbab I. Arbab (1998) *Gen. Rel. Grav.* **30**, 1401.

[12] Arbab I. Arbab, *gr-qc*/9909044.

[13] Arbab I. Arbab (2001), in *5th RESCEU International Symposium on New Trends in Theoretical and Observations Cosmology*, Japan, 2001, published in '*Tokyo 2001, New trends in theoretical and observational cosmology*', 281.

[14] Arbab I. Arbab (2002) *Grav. Cosmol.* **8**, 227.

[15] Arbab I. Arbab (2003) *Class. Quantum Grav.* **20**, 93.

[16] Arbab I. Arbab (2004) *Astrophys. Space Sci.* **291**, 141.

[17] Armendariz-Picon, C., Mukhanov, V. and Steinhardt, P. J. (2001) *Phys. Rev. D* **63**, 103510.

[18] Bachall, N. A. , Ostriker, J. P. , Perlmutter, S. and Steinhardt, P. J. (1999), *Science* **284**, 1481.

[19] Bagla, J. S., Jassal, H. K. and Padmanabhan, T. (2003) *Phys. Rev. D* **67**, 063504.

[20] Banerjee, A., Dutta Chaudhuri, S. B. and Banerjee, N. (1985) *Phys. Rev. D* **32**, 3096.

[21] Banerjee, N. and Pavon, D. (2001), *Phys. Rev. D* **63**, 043504; *Class. Quantum Grav.* **18** 593.

[22] Barreiro, T., Copeland, E. J. and Nunes, N. J. (2000) *Phys. Rev. D* **61**, 127301.

[23] Battye, R. A. , Bucher, M. and Spergel, D. (1999), "*Domain wall dominated universe*", *astro-ph*/990847.

[24] Beesham, A. (1993) *Phys. Rev. D* **48**, 3539.

[25] Beesham, A. (1994) *Gen. Rel. Grav.* **26**, 159.

[26] Benaoum, H. B., *hep-th*/0205140.

[27] Bennett, C. L. (2006), *Nature* **440**, 1126.

[28] Bento, M. C., Bertolami, O. and Sen, A. A. (2002) *Phys. Rev. D* **66**, 043507.

[29] Bento, M. C., Bertolami, O. and Sen, A. A. (2003) *Phys. Lett. B* **575**, 172.

[30] Berger, M. S. and Shojaei, H. (2006) *Phys. Rev. D* **74**, 043530.

[31] Bergmann, P. G. (1968) *Int. J. Phys.* **1**, 25.

[32] Berman, M. S. and Som, M. M. (1990) *Int. J. Theor. Phys.* **29**, 1411.

[33] Berman, M. S. (1991) *Phys. Rev. D* **43**, 1075.

[34] Bertolami, O. (1986) *Nuovo Cimento* **93B**, 36.

[35] Bertolami, O. (1986) *Fortschr. Phys.* **34**, 829.

[36] Bertolami, O. and Martins, P. J. (2000), *Phys. Rev. D* **61**, 064007.

[37] Bilic, N., Tupper, G. B. and Viollier, R. D. (2002) *Phys. Lett. B* **535**, 17.

[38] Bondi, H. (1960), *Cosmology (Cambridge monographs on physics)*, Cambridge University Press.

[39] Bonanno, A. and Reuter, M. (2002) *Phys. Lett. B*, **527**, 9.

[40] Brans, C. and Dicke, R. (1961), *Phys. Rev.* **124**, 925.

[41] Brax, P. and Martin, J. (1999) *Phys. Lett. B* **468**, 40.

[42] Brax, P. and Martin, J. (2000) *Phys. Rev. D* **61**, 103502.

[43] Brevik, I., Nojiri, S., Odintsov, S. D. and Vanzo, L. (2004) *Phys. Rev. D* **70**, 043520.

[44] Brevik, I., Gorbunova, O. G. and Timoshkin, A. V. (2007) *Eur. Phys. J. C* **51**, 179.

[45] Brustein, R. and Hadad, M. (1999) *Phys. Rev. Lett.* **82**, 3016.

[46] Bucher, M. and Spergel, D. (1999), *Phys. Rev. D* **60**, 043505.

[47] Cai, R. G. and Wang, A. (2005) *JCAP* **03**, 002.

[48] Caldwell, R. R. , Dave, R. and Steinhardt, P. J. (1998), *Phys. Rev. Lett.* **80**, 1582.

[49] Canuto, V., Hsieh, S. H. and Adams, P. J. (1977) *Phys. Rev. Lett.* **39**, 429.

[50] Chaplygin, S. (1904) *Sci. Mem. Moscow Univ. Math. Phys.* **21**, 1.

[51] Calcagni, G. and Liddle, A. R. (2006) *Phys. Rev. D* **74**, 043528.

[52] Capozziello, S., Cardone, V., Elizalde, E., Nojiri, S. and Odintsov, S. D. (2006) *Phys. Rev. D* **73**, 043512.

[53] Cardenas, V. H. and Campo, S. D. (2004) *Phys. Rev. D* **69**, 083508.

[54] Carroll, S. M., Press, W. H. and Turner, E. L. (1992) *Ann. Rev. Astron. Astrophys.* **30**, 499.

[55] Carvalho, J. C., Lima, J. A. S. and Waga, L. (1992) *Phys. Rev. D* **46**, 2404.

[56] Chen, W. and Wu, Y. S. (1990) *Phys. Rev. D* **41**, 695.

[57] Chiba, T. (2003) *Phys. Lett. B* **575**,1.

[58] Chimento, L. P. (2004) *Phys. Rev. D* **69**, 123517.

[59] Chimento, L. P., Lazkoz, R. (2005) *Phys. Lett. B* **615**, 146.

[60] Cimento, L. P. , Jakubi, A. S. and Pavon, D. (2000), *Phys. Rev. D* **62**, 063508.

[61] Cohn, J. D. (1998) *Astrophys. J. Suppl.* **259**, 213.

[62] Colistete Jr., R., Fabris, J. C., Goncalves, S. V. B. and de Souza, P. E. (2004) *Int. J. Mod. Phys. D* **13**, 669, *astro-ph*/0303338.

[63] Collins, C. A., Nichol, R. C. and Lumsden, S. L. (1992), *Mon. Not. Roy. Astro. Soc.* **254**, 295.

[64] Copeland, E. J., Liddle, A. R. and Wands. (1998) *Phys. Rev D* **57**, 4686.

[65] Copeland, E. J., Garousi, M. R., Sami, M. and Tsujikawa, S. (2005) *Phys. Rev D* **71**, 043003.

[66] Copeland, E. J., Sami, M. and Tsujikawa, S. (2006) *Int. J. Mod. Phys. D* **15** 1753.

[67] Das, A., Gupta, S., Saini, T. D. and Kar, S. (2005) *Phys. Rev D* **72** 043528.

[68] DeBenedictis, A., Das, A. and Kloster, S. (2004) *Gen. Rel. Grav.* **36**, 2481.

[69] Debnath, U., Banerjee, A. and Chakraborty, S. (2004) *Class. Quantum Grav.* **21**, 5609.

[70] Debnath, U. (2007) *Astrophys. Space Sci.* **312**, 295.

[71] Dirac, P. A. M. (1938), *Proc. R. Soc. A* **165**, 119.

[72] Dirac, P. A. M. (1973), *Proc. R. Soc. A* **333**, 403.

[73] Dirac, P. A. M. (1975), '*The General Theory of Relativity* , New York : Wiley.

[74] Dirac, P. A. M. (1979), *Proc. R. Soc. A* **365**, 19.

[75] Dodelson, S. , Kaplinght, M. and Stewart, E. (2000), "*Tracking oscillating energy*", *astro-ph*/0002360.

[76] Elizalde, E. , Nojiri, S., Odintsov, S. D. and Wang, P. (2005) *Phys. Rev. D* **71**, 103504.

[77] Ellis, G. F. R. and Madsen, M. S. (1991) *Class. Quantum Grav.* **8** 667.

[78] Ellis, G. F. R. and Wainwright, J. (1997), in '*Dynamical systems in Cosmology*', edited by J. Wainwright and G. F. R. Ellis (Cambridge Univ. Press, Cambridge, England).

[79] Endo, M. and Fukui, T. (1977) *Gen. Rel. Grav.* **8**, 833.

[80] Fabris, J. C., Gonsalves, S. V. B. and de Souza, P. E. (2002) *Gen. Rel. Grav.* **34**, 53.

[81] Faraoni, V. (2000), *Phys. Rev. D* **62**, 023504.

[82] Ferreira, P. G. and Joyce, M. (1998), *Phys. Rev. D* **58**, 023503.

[83] Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A. and Huchra, J. P. (1993), *Astrophys. J.* **402**, 42.

[84] Freese, K., Adams, F. C., Freeman, J. A. and Mottola, E. (1987) *Nucl. Phys. B.* **287**, 797.

[85] Frieman, J. A. (1993) *Lectures given at Baksan International School on Particle Physics and Cosmology, Baksan, Russia* and *TASI'93, Boulder, Colo*, published in *Baksan Particle Phys.*, 135, *astro-ph/9404040.*

[86] Frieman, J., Hill, C. T., Stebbins, A. and Waga, I., (1995) *Phys. Rev. Lett.* **75**, 2077.

[87] Frieman, J. A., Turner, M. and Huterer, D. (2008) *Ann. Rev. Astron. Astrophys* **46**, 385.

[88] Frolov, A., Kofman, L. and Starobinsky, A. (2002) *Phys. Lett. B* **545**, 8.

[89] Garfinkle, D. (2006) *Class. Quantum Grav.* **23**, 4811.

[90] Garnavich, P. M. et al (1998), *Astrophys. J.* **509**, 74.

[91] Gasperini, M. (1987) *Phys. Lett. B* **194**, 347.

[92] Gasperini, M. (1988) *Class. Quantum Grav.* **5**, 521.

[93] Geller, M. and Huchra, J. (1989) *Science* **246**, 897.

[94] Gibbons, G. W. (2002) *Phys. Lett. B* **537**, 1.

[95] Gibbons, G. W. (2003), lecture given at *Workshop on the Quantum Structure of Space-time and the Geometrical Nature of the Fundamental Interactions*, Belgium, 2002, *Class. Quant. Grav.* **20**, S321.

[96] González-Diaz, P. F. (2003) *Phys. Rev. D* **68**, 021303 (R).

[97] Gorini, V., Kamenshchik, A., Moschella, U. and Pasquier, V., (2003) in '*the proceedings of 10th Marcel Grossmann Meeting on Recent Developments in Theoretical and Experimental General Relativity, Gravitation and Relativistic Field Theories*', Brazil, 2003, in '*Rio de Janeiro 2003, Recent developments in theoretical and experimental general relativity, gravitation, and relativistic field theories, pt. B*', 840-859.

[98] Gorini, V., Kamenshchik, A. and Moschella, U. (2003), *Phys. Rev. D* **67**, 063509.

[99] Gott, J., R. III and Rees, M., J. (1987) *Mon. Not. R. Astron. Soc.* **227**, 453.

[100] Guo, Z. K. and Zhang, Y. Z. *astro-ph*/0509790.

[101] Guo, Z. K. and Zhang, Y. Z. (2007) *Phys. Lett. B* **645**, 326.

[102] Herrera, R., Pavon, D. and Zimdahl, W. (2004) *gen. Rel. Grav.* **36**, 2161.

[103] Hu, B. and Ling, Y. (2006) *Phys. Rev. D* **73**, 123510.

[104] Hwang, J. C. and Noh. H (2001), *Phys. Rev. D* **64**, 103509.

[105] Jackiw, R. (2000), *A Particle Field Theorist's Lectures on Supersymmetric, Non-Abelian Fluid Mechanics and d-Branes*, a series of six lectures given at the Centre de Recherches Mathematiques in Montreal, 2000,*physics*/0010042v1.

[106] Kalligas, D., Wesson, P. and Everitt, C. W. F. (1992) *Gen. Rel. Grav.* **24**, 351.

[107] Kalligas, D., Wesson, P. and Everitt, C. W. F. (1995) *Gen. Rel. Grav.* **27**, 645.

[108] Kamenshchik, A., Moschella, U. and Pasquier, V. (2001), *Phys. Lett. B* **511**, 265.

[109] Karman, T. von (1941) *J. Aeron. Sci.* **8**, 337.

[110] Kolb, E. W. (1989) *Astrophys. J.* **344**, 543.

[111] Lima, J. A. S. and Carvalho, J. C. (1994) *Gen. Rel. Grav.* **26**, 909.

[112] Lima, J. A. S. and Maia, J. M. F. (1994) *Phys. Rev. D* **49**, 5597.

[113] Lima, J. A. S. and Trodden, M. (1996) *Phys. Rev. D* **53**, 4280.

[114] Liu, Dao-Jun and Li, Xin-Zhou (2005) *Chin. Phys. Lett.* **22**, 1600.

[115] Lopez, J. L. and Nanopoulos, D. V. (1996) *Mod. Phys. Lett.* **A11**, 1.

[116] Lu, J., Xu, L., Li, J., Chang, B., Gui, Y. and Liu, H. (2008) *Phys. Lett. B* **662**, 87.

[117] Maartens R. (2004), *Living Rev. Rel.* **7** 7.

[118] Maddox, S. J., Efstathiou, G. and Sutherland, W. J. (1990), *Mon. Not. Roy. Astron. Soc.* **246**, 433.

[119] Makler, M., de Oliveira, S. Q. and Waga, I. (2003) *Phys. Lett. B* **555**, 1.

[120] Maia, M. D. and Silva, G. S. (1994) *Phys. Rev. D* **50**, 7233.

[121] Masso, E. (2003) *Nucl. Phys. Proc. Suppl.* **114**, 67.

[122] Matyjasek, J. (1995) *Phys. Rev. D* **51**, 4154.

[123] McCrea, W. H. (1971) *Quarterly J. Roy. Astron. Soc.* **12**, 140.

[124] Narlikar, J. V. (2002), '*An Introduction to Cosmology*', Foreword by Sir F. Hoyle, Cambridge University Press.

[125] Nojiri, S. and Odintsov, S. D. (2005) *Phys. Rev. D* **72**, 023003.

[126] Nojiri, S. and Odintsov, S. D. (2006) *Phys. Rev. D* **639**, 144.

[127] Nordtvedt Jr., K., (1970) *Astrophys. J* **161**, 1059.

[128] Olson, T. S. and Jordan, T. F. (1987) *Phys. Rev. D* **35**, 3258.

[129] Ostriker, J. P. and Steinhardt, P. J. (1995), *Nature* (London) **377**, 600.

[130] Ott. T. (2001), *Phys. Rev. D* **64**, 023518.

[131] Ozer, M. and Taha, M. O. (1986) *Phys. Lett. B* **171**, 363.

[132] Ozer, M. and Taha, M. O. (1987) *Nucl. Phys. B.* **287**, 776.

[133] Overduin, J. M. and Cooperstock, F. I. (1998) *Phys. Rev. D* 5**8**, 043506.

[134] Padmanabhan. T. (2002), *Phys. Rev. D* **66**, 021301 **R**.

[135] Padmanabhan. T. (2003), *Phys. Rept.* **380,** 235.

[136] Padmanabhan. T. (2006), plenary talk at *Albert Einstein's Century International Conference*, France, 2005, published in *AIP Conf. Proc.* **861**, 179, '*Paris 2005, Albert Einstein's century*', 179, *astro-ph*/0603114.

[137] Paul, B. C. and Paul, D. (2005) *Int. J. Mod. Phys. D* **14**, 1831.

[138] Pavon, D. (1991) *Phys. Rev. D* **43**, 375.

[139] Peebles, P. J. E. (1984), *Astrophys. J.* **284**, 439.

[140] Peebles, P. J. E. and Ratra, B. (1988), *Astrophys. J. Lett.* **325**, L17.

[141] Peebles, P. J. E. and Ratra, B. (2002), *astro-ph*/0207347.

[142] Peebles, P. J. E. and Ratra, B. (2003), *Rev. Mod. Phys.* **75**, 559 .

[143] Perlmutter, S. et al (1998), *Nature* (London) **391**, 51.

[144] Perlmutter, S. et al (1999), *Astrophys. J.* **517**, 565.

[145] Perlmutter, S. , Turner, M. S. and White, M. (1999), *Phys. Rev. Lett.* **83**, 670.

[146] Ratra, B. and Peebles, P. J. E. (1988), *Phys. Rev. D* **37**, 3406.

[147] Riess, A. G. et al (1998), *Astrophys. J.* **116**, 1009.

[148] Roszkowski, L. (1999), invited review talk at *Conference on Particle Physics and the Early Universe (COSMO 98)*, Monterey, CA, 1998, published in '*Asilomar 1998, Particle physics and the early universe*', 316, *hep-ph*/9903467.

[149] Sahni, V. and Starobinsky, A. A. (2000), *Int. J. Mod. Phys. D* **9**, 373.

[150] Sahni, V. and Wang, L. (2000) *Phys. Rev. D* **62**, 103517.

[151] Sahni, V., Saini, T. D., Starobinsky, A. A. and Alam, U. (2003), *JETP Lett.* **77**, 201.

[152] Sahni, V. (2004), Lectures given at *2nd Aegean Summer School on the Early Universe*, Greece, 2003, in *Lect. Notes Phys.* **653** 141; '*Ermoupoli 2003, The physics of the early universe*', 141; it astro-ph/0403324.

[153] Sahoo, B. K. and Singh, L. P. (2003) *Modern Phys. Lett. A* **18**, 2725.

[154] Sami, M. (2003) *Mod. Phys. Lett. A* **18**, 691.

[155] Sen, S. and Sen, A. A.(2001) *Phys. Rev. D* **63**, 124006.

[156] Sen, A. (2002) *JHEP* **04**, 048.

[157] Sen, A. (2002) *JHEP* **07**, 065.

[158] Sen, A. (2002) *Mod. Phys. Lett. A* **17**, 1797.

[159] Sen, A. (2003) *Int. J. Mod. Phys. A* **18**, 4869.

[160] Sethi, G., Singh, S. K., Kumar, P., Jain, D. and Dev, A. (2006) *Int. J. Mod. Phys. D* **15**, 1089.

[161] Shao, Y. Gui, Y. (2008) *Mod. Phys. Lett. A* **23**, 65.

[162] Shiu, G. and Wasserman, I. (2002) *Phys. Lett. B* **541**, 6.

[163] Silveira, V. and Waga, I. (1994) *Phys. Rev. D* **50**, 4890.

[164] Silveira, V. and Waga, I. (1997) *Phys. Rev. D* **56**, 4625.

[165] Smolin, L. (1979) *Nucl. Phys. B* **160**, 253

[166] Smoot, G. et al (1992) *Astrophys. J.* **396**, L1.

[167] Springel, V., Frenk, C. S., White, S. D. M. (2006) *Nature* **440**, 1137.

[168] Srivastava, S. K. (2004) *gr-qc*/0411088, *gr-qc*/0409074,

[169] Stanyukovich, K. (1960) '*Unsteady Motion of Continuos Media*', Pergamon, Oxford, UK.

[170] Stefancic, H. (2005) *Phys. Rev. D* **71**, 084024.

[171] Torres, L. F. B. and Waga, I. (1996) *Mon. Not. R. Astron. Soc.* **279**, 712.

[172] Tsien, H. S. (1939) *J. Aeron. Sci.* **6**, 399.

[173] Urena-Lopez, L. A. and Matos, T. (2000) *Phys. Rev. D* **62**, 081302.

[174] Vishwakarma, R. G. (2000) *Class. Quant. Grav.* **17**, 3833.

[175] Vishwakarma, R. G. (2005) *Gen. Rel. Grav.* **37**, 1305.

[176] Wagoner, R. V. (1970) *Phys. Rev. D* **1**, 3209.

[177] Wang, L. , Caldwell, R. , Ostriker, J. P. and Steinhardt, P. J. (2000), *Astrophys. J.* **530**, 17.

[178] Weinberg, S. (1989), *Rev. Mod. Phys.* **61**, 1.

[179] Weinberg, S. (1972), '*Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*', Wiley

[180] Wesson, P. S. (1978), '*Cosmology and Geophysics*', Oxford : Oxford University Press.

[181] Wesson, P. S. (1980), '*Gravity,Particles and Astrophysics*', Dordrecht : Rieded.

[182] Wetterich, C. (1988) *Nucl. Phys. B* **302**, 668.

[183] Wetterich, C. (1995) *Astron. Astrophys.* **301**, 321.

[184] Zee, A. (1979) *Phys. rev. Lett.* **42**, 417.

[185] Zel'dovich, Y., B. (1967) *JETP Lett* **6**, 316, *Pisma Zh. Eksp. Teor. Fiz.* **6**, 883.

[186] Zel'dovich, Y., B. (1968) *Soviet Phys. Uspekhi* **11**, 381.

[187] Zimdahl, W. (2006) *Int. J. Mod. Phys. D* **14**, 2319.

[188] Zlatev, I., Wang, L. and Steinhardt, P.J. (1999), *Phys. Rev. Lett.* **82**, 896.

[189] http://cosmology.berkeley.edu